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Medicine. — *The ultimate structure of the striped muscle fibre of the frog distinguishable with the microscope.* By J. W. LANGELAAN.

(Communicated at the meeting of November 29, 1941.)

On a previous occasion I gave a short account of my researches on the structure of the muscle fibre, and especially drew your attention to an interferential pattern caused by a fresh and structurally intact fibre. This pattern consists of a fine, narrow striation of the *q*-stripe. White dashes alternate regularly with small dark strokes. The period of this pattern approximately amounts to 0.7 micron. The white dashes represent the light maxima, the dark strokes the minima of the interference figure. The *I*-stripe presents no recognisable structure with the interferential arrangement of the microscope. The interference pattern forms the only indication of the existence of a regular structure of the fibre, of which the characteristic dimensions are commensurable with the wave-length of the visible light.

The interferential method, by which the pattern can be rendered visible, forms a strict application of ABBE's theory on microscopical vision. This spectrum theory has been laid down in a compendious, mathematical form by LUMMER and REICHE [1]). Unfortunately these authors dealt only with the problem of twodimensional gratings, but the muscle fibre is a tridimensional one. The method consists of the appliance of a narrow incident beam of which the divergence angle is equal to, or smaller than, 3°, monochromatic light, and a darkfield objective-lens according to the method of SPIERER [2]). The small opaque shield upon the curved surface of the front lens of the objective screens off the central bundle (spectrum of zero order) of the incident beam diffracted at the object. In this way the darkfeld is made. An adjustible diaphragm, forming the exit-pupil of the objective, makes it possible to restrict the number of the spectra which co-operate in the formation of the interferential pattern.

The analysis of the primary interferential image, present in the neighbourhood of the principal focal plane of the objective, indicates that the interferential pattern, called forth by the microscopic structure of the muscle fibre, arises from the joint effect of two intercrossing gratings. The elements of which these patterns are composed run in a diagonal direction in respect of the length of the fibre. They intercross at an angle varying between 60° and 80°. When the interferential pattern is screened off by a diaphragm, placed in the neighbourhood of the principal focal plane of the objective, a vague image remains of the ordinary cross striation of the muscle fibre. The diameter of the alternating light and dark stripes amounts to about 1 micron. This image becomes more distinctly visible when the divergence of the incident beam is increased and the exit diaphragm of the objective widely opened. This image obviously results from a periodic variation in the density of the fibre, causing a variation in the permeability of the tissue to light. The structure of this differentiation is submicroscopic, for it cannot be distinguished with the microscope, nor does it give rise to observable interferences.

A method, not essentially new, renders it possible to convert a plastic muscle fibre into a brittle one without materially disturbing the microscopical structure of the fibre nor its characteristic dimensions. For this purpose a fresh and structurally intact sartorius muscle, extended to its natural length, is put into an aqueous solution of chromalum of 3 to 5 percent. In this solution the plastic fibres become hard and brittle in the course of about two months. If, now, one tries to cut the muscle on an ice-microtome, the fibres are crushed.

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[1]) LUMMER und REICHE, Lehre v. d. Bildentstehung i. Mikroskop, 1910.

[2]) SPIERER, Ch. Koll. Zeits. **51** (1), 162—163 (1930); **53** (1) 88—90 (1930).

by the knife and burst. This always happens independently of the direction in which the muscle is cut.

The fragments, resulting from the bursting of the fibres, prove to be split off along preformed cleavage planes, in the same way as a brittle crystal would do under similar circumstances. Most of the fragments split off along a plane parallel to the surface of the fibre. This plane represents the tangential cleavage plane. The smaller side of these fragments is formed by the radial cleavage plane, whilst the third side is formed by the transverse, or BOWMAN, cleavage plane. The directions of these planes are determined by the internal structure of the fibre. They represent the faces of the structure. The configuration of the fragments indicates that they have split off from an orthogonal grating. The measurement of the length of the sides of the fragments proves that they are all even multiples of a unit length of about 0.5 micron. This dimension represents the diameter of a single structural leaf of the grating. It follows from this observation that the layers of which the grating has been built up are each composed of two leaves. Near the margin of the fragments small zones often occur, of which the diameter is about 0.5 micron. These thin, single-leaved edges arise through the bursting of the fibre when crushed by the knife. I never deliberately succeeded by means of needles or other mechanical contrivances in splitting up a layer into its leaves.

The single-leaved edges present no distinguishable structure when observed by means of white light and the image-forming arrangement of the microscope. Photos, however, made by means of violet light ($\lambda = 445$ millimicra) and an incident beam with a rather small divergence angle, show the structure of the leaf. The image present on the photographic plate consists of two sets of point rows, intercrossing at right angles. The points are equidistant, having in both directions a period of about 0.5 micron. The same observation can be made on the radial face of the fragments. The points visible on the photos are diffraction (AIRY) disks. They represent therefore the images of some submicroscopical detail, made self-luminous by the light of the incident beam. The real structure of the detail remains, of course, undetermined.

The photograms can be measured, and prove that the period of the point row in the longitudinal direction of the fibre amounts to 0.55 micron. The period in the tangential and radial directions varies round 0.45 micron. The arithmetical means of the measurements in these directions indicate, however, that the tangential period is slightly greater than this figure, and the radial period slightly less. This may be indicated by writing for the tangential period 0.45^5 and for the radial period the figure 0.44^5. The basic cell of the microscopic grating is, therefore, a simple rhombic prism, closely approaching a tetragonal one. It must, however, be emphasized, that this represents the basic cell of the contractile part of the fibre for the basic cell of the sarcoplasmatic part is slightly more complicated.

The description of the structure of the microscopic grating has been based upon images presented by a single structural leaf, for this excludes axial interferences. As far as I know axial interferences have always been neglected notwithstanding that they may produce interferences under conditions, making interference entirely unexpected. For that reason axial interferences superposed upon a microscopic image are not recognised. Lateral interferences are well known and also treated theoretically, though their effects often fail to be recognised [3]).

Experiments made by means of an appropriate buffer-solution, render it possible to follow the desintegration of the microscopic grating for several hours. In this solution the traces of but one element becomes visible, when using a paraboloid darkfield condensor. The only element which becomes observable in this buffer is a short thread-like fibril of which the characteristic diameter is submicroscopic. For that reason it cannot be seen with the image-forming constellation. A great number of these fibrils, still bearing their negative electric charges, adhere to the coverglass, other ones, after having lost their

[3]) RAYLEIGH, Scientific Papers, IV (222), 235—260 (1903).

charges, cling together and form small, irregularly shaped granules. The largest of these granules are of microscopic dimensions. The fibrils, adherent to the coverglass, execute brownian, pendulating movements. The similarity in shape and dimension of the diffraction figures, caused by the moving fibrils, render it probable that they have all approximately the same length and diameter.

The most probable conclusion to be drawn from these experiments is, now, that the muscle fibre forms a tridimensional, rhombic grating, closely approaching a tetragonal grating. The dimensions of the three periods, characterising the grating, approach the limit of the resolving power of the objective-lenses. The elements composing the grating are most probably thread-like filaments. The submicroscopic knot-points of this filamentous grating can be made self-luminous. These self-luminous points appear in the image as the diffraction disks, composing the orthogonal point-rows observable in the photograms made by means of violet light and the image-forming arrangement.

Comparative Physiology. — *Ueber den Tonus des Schneckenfusses (Helix pomatia L.).* III. *Zwei antagonistische Zentren in den Pedalganglien.* (Vorläufige Mitteilung.) Von H. J. JORDAN und N. POSTMA[1]). (Aus dem Institut für vergleichende Physiologie der Universität zu Utrecht.)

(Communicated at the meeting of November 29, 1941.)

A. Einleitung.

Auf Grund seiner Erfahrungen an Aplysia war JORDAN (1) zur Ueberzeugung gekommen, dass die Pedalganglien den Tonus der Muskulatur dieser Schnecken lediglich durch Hemmung (und „Lösung") regulieren. Er verfügte über keine Versuchsergebnisse, die ihn zwangen, neben einem hemmenden, die Existenz eines erregenden Zentrums in diesen Ganglien anzunehmen. Exstirpation der Ganglien hat stets starke Steigerung des Muskeltonus zur Folge.

Wenn man eine Aplysia eine Nachtlang in den Eisschrank bringt, so findet man das Tier am folgenden Morgen vollkommen erschlafft, die Muskulatur tonuslos. Bringt man das Tier nun in zimmerwarmes Seewasser, so tritt tonische Verkürzung auf und nach einigen Minuten ist es wieder normal. Macht man den Versuch mit einer Aplysia, der die Ganglien exstirpiert wurden, so verläuft der Prozess des Tonusverlustes im Eisschrank wie beim normalen Tiere. Die Wiederherstellung des Tonus im zimmerwarmen Wasser dahingegen erfolgt in ungemein übertriebenem Ausmass: das Tier schrumpft zu einem harten unförmigen Klumpen (vergl. 2). Es war JORDAN nie gelungen durch Reizung der Pedalganglien und der diese Zentren mit den Muskeln verbindenden Nerven „Tonuslösung" zu erhalten.

Schwache Kokainisierung der Pedalganglien aber verursacht Tonuslösung in den Parapodien. Kokainisiert man eine Tierhälfte, so fällt auch der Tonus in der anderen, die mit der kokainisierten Hälfte nur durch das zentrale Nervensystem verbunden ist.

All das machte den Eindruck, als ob die Regulierung auf dem Prinzipe der „Isostasie" beruhe, die sich am allgemeinsten in der Gleichheit des Tonus innerhalb der Gesamtmuskulatur offenbart. Es könnte sich daher um eine Ausgleichung des Aktivitätszustandes in allen Teilen (Muskeln und Ganglien) handeln, deren Mechanik natürlich unbekannt bleibt. Prinzipiell an dieser Form, die Erscheinungen zu beschreiben, ist die Auffassung, dass die Pedalganglien für den Tonus *ein einheitliches Zentrum* sind, und eine Differenzierung zwischen erregendem und hemmendem Zentrum in diesen Ganglien fehlt.

Diese Einheitlichkeitshypothese wurde später zur Beschreibung der Ergebnisse ausgebaut, die JORDAN bei seinen Untersuchungen an Helix pomatia erhalten hatte (Siehe II, S. 1244). Mittlerweile haben POSTMA und MAAS die Möglichkeit gefunden, an ein und demselben Helixfuss zahlreiche Wiederholungskurven zu erhalten (6 u. 5). Nach jeder Dehnung wird der Fuss wieder zur Ausgangslänge gebracht und er lässt sich mit der gleichen Gesetzmässigkeit wie die „erste oder ursprüngliche Dehnungskurve" durch „indifferente" Last wieder dehnen.

Mit Hülfe dieser Wiederholungskurven gelang es POSTMA (6 u. 7) eine feste Norm für das Niveau der Pedalkonstanz zu gewinnen. Bei der Präparation wird der Fuss immer etwas gedehnt; in diesem gespannten Zustand werden die Ganglien für die kommenden Eingriffe freigelegt. Wenn man nun, ohne jeden weiteren Eingriff, den Fuss an den Ordinatenschreiber des Kymographions befestigt und mit einem bestimmten „indifferenten" Gewicht belastet, so tritt die typische pedaltonische Dehnungskurve auf und zwar mit einer Pedalkonstanz, die bei der „Spannungslänge" der Präparation liegt und bei richtiger

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[1]) Die Versuche wurden durch N. POSTMA, mit Unterstützung der Niederl. Regierung aus den Einkünften der „Zomerpostzegels", ausgeführt.

Wahl des Gewichtes, in den aufeinanderfolgenden Wiederholungskurven unverändert bleibt. Das „indifferente" Gewicht muss für jedes Objekt bestimmt werden. Die Spannungslänge ist ein festes Mass für die Einflüsse der Pedalganglien. Wenn man ein zu niedriges Gewicht wählt, so steigt in den aufeinanderfolgenden Wiederholungskurven das Niveau allmählich, während es ebenso allmählich absinkt, wenn das Gewicht höher ist als das indifferente.

JORDAN hatte seine Eingriffe immer erst nach Erreichen der Pedalkonstanz ausgeführt. Den Einfluss dieser Eingriffe auf die erste steile Strecke der pedaltonischen Dehnungskurve hatte er daher nicht beobachtet. Dies hat nun POSTMA (8, Mitteilung II) getan und neue, für unser Verständnis der zentralen Tonusregulierung wichtige Resultate erhalten. Im Laufe dieser Untersuchungen ergaben sich zahlreiche Erscheinungen, die es immer schwieriger machten, an der Einheitlichkeit der Ganglien fest zu halten, Erscheinungen, die POSTMA am Schluss seiner Mitteilung II zusammengefasst hat. Auch die Arbeiten über den Einfluss des Aktivitätszustandes der Ganglien auf den Tonus (von JORDAN, Frl. SLUITER und POSTMA) und auf die Regulierung der Herztätigkeit (von N. v. TIEL), beide bei Helix pomatia, hatten einige Resultate ergeben, die auf *das Vorhandensein echter Hemmungszentren und hemmender Impulse hinwiesen,* sodass wir schon zu Anfang unserer Untersuchungen nicht zweifelten, dass wir zu einer neuen Auffassung des zentralen Geschehens bei unserm Objekte würden kommen müssen.

Unsere häufig wiederholten Versuche mit partieller Vergiftigung der Ganglien mit Kokain und mit Kochsalz haben bei Helix niemals diejenige Reproduzierbarkeit gezeigt, die notwendig gewesen wäre, um eine feinere Analyse der Ganglienbezirke und ihrer vermuteten spezifischen Funktionen durchzuführen. Wir entschlossen uns zu versuchen, ob wir, an Stelle von unkontrollierbar diffundierenden Chemikalien, den konstanten Strom in einer Stärke verwenden könnten, in welcher er *keine Reizung, wohl aber Elektrotonus* verursacht. Denn wir wussten, dass Reizung und Veränderung des Aktivitätszustandes der Ganglien verschiedene Resultate haben. Die einfache Technik, vor allem die Form der unpolarisierbaren Chlorsilberelektroden, wird in der ausführlichen Publikation beschrieben werden. Hier sei nur vorweg genommen, dass die infraoesophageale Ganglienmasse zwischen beide Elektroden kommt, sodass die eine dorsal, die andere ventral liegt. Durch einen Umpoler kann man den Katelektrotonus entweder dorsal oder ventral erzeugen. Versuche mit einer indifferenten Elektrode sind noch im Gang.

Zu den ältern Versuchen, in welchen Kokain oder Kochsalz immer dorsal appliziert worden war, wurden neue hinzugefügt, mit *ventraler Applikation der genannten Chemikalien.* Dadurch konnten die widersprechenden Resultate älterer Untersuchungen aufgeklärt und die Ergebnisse der elektrotonischen Versuche bestätigt werden.

Weiterhin wurden die Versuche von POSTMA (8) mit mechanischer Reizung von Ganglien und Nerven wiederholt, *unter Anwendung von Einzelöffnungsschlägen und genauer Abstufung der Reizstärke.*

B. Ergebnisse.

I. *Versuche mit konstantem Strom.*

Der Kürze halber sprechen wir bei Versuchen, bei denen die dorsale Elektrode die Kathode des subliminalen konstanten Stromes war, von *dorsalem Strom.* Lag dahingegen die Kathode ventral, so reden wir von *ventralem Strom.* Es hat sich als vorteilhaft erwiesen, unterhalb der Stromstärke zu bleiben, bei welcher Schluss und Oeffnung Zuckung erzeugen würden.

1. *Veränderung des Aktivitätszustandes in den Pedalganglien durch den Elektrotonus.*

Der *dorsale Strom erhöht* als primäre Reaktion das Niveau der Pedalkonstanz, das in Vorversuchen, zugleich mit der Bestimmung der indifferenten Last, festgestellt worden war. Der *ventrale Strom erniedrigt* dahingegen diese Konstanz in aufeinanderfolgenden Wieder-

holungskurven (Abb. 1). Diese Versuche sind sehr reproduzierbar (100 %). Es ist gleichgültig ob man die Kathode erst dorsal schaltet und daraufhin umpolt oder umgekehrt. Offenbar enthalten die Pedalganglien **zwei topographisch geschiedene Zentra**. Wie

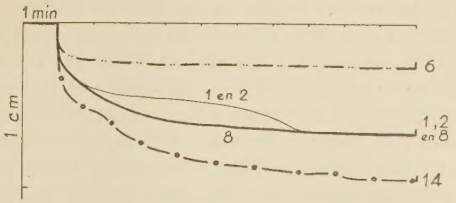

Abb. 1. Wiederholungskurven von einem Schneckenfuss im Zusammenhang mit den Pedalganglien. Dehnungskurven mit 10 g Last von 10 Minuten Dauer. Ordinaten: effektive Längenänderung des Fusses (Ordinatenschreiber nach N. POSTMA, 1933); Abszissen: die Zeit. Versuche mit konstantem Strom (Electrotonus). 1 und 2: Vorversuche, ohne Strom. Von Kurve 5 an (nicht gezeichnet) Strom geschlossen, dorsale Elektrode ist Kathode. 6: Niveausteigerung durch dorsalen Strom; am Ende dieser Dehnung wird der Strom geöffnet. 8: ohne Strom; vor Kurve 11 (nicht gezeichnet) Stromschluss, ventrale Elektrode ist nunmehr die Kathode. 14: Niveauverlust durch ventralen Strom.

N. V. TIEL (9) für die extracardiale Regulierung des Helixherzens in den Visceralganglien ein erregendes und ein hemmendes Zentrum fand, so ergab sich aus unsern vorliegenden Versuchen die Existenz eines erregenden und eines hemmenden Tonuszentrums, welch letzteres, bei Erhöhung seines Aktivitätszustandes, das Niveau der Pedalkonstanz erniedrigt. Damit fällt die Möglichkeit, den Gesamtkomplex der Erscheinungen mit dem Schema der einheitlichen Ganglienwirkung auf Grund des Ausgleichsprinzips zu beschreiben. Die Eindeutigkeit dieser Versuche beruht offenbar auf dem Gegensatz zwischen Katelektrotonus und Anelektrotonus, wodurch eine gleichzeitige Aktivitätserhöhung in beiden antagonistischen Zentren ausgeschlossen ist. Bei unseren früheren Versuchen über Aktivitätserhöhung und -erniedrigung mit Kochsalz und Kokain war das, der Diffusion dieser Stoffe wegen, natürlich anders.

2. *Summation verschiedener Einflüsse auf die Konstanz.*

Wie erwähnt steigt unter Einwirkung geringer Last das Niveau der Pedalkonstanz in aufeinanderfolgenden Wiederholungskurven (6 u. 7). Dieser Anstieg unterbleibt bei gleichzeitiger Einschaltung des ventralen Stromes. Wiederholungskurven mit hoher Last zeigen zunehmenden Niveauverlust der Pedalkonstanz, der durch dorsalen Strom neutralisiert wird. Gleichsinnige Einflüsse unterstützen einander. Wenn bei einem bestimmten Fuss bei einem Gewicht von 10 g das Niveau der Pedalkonstanz oberhalb der Normalen liegt, d.h. 10 g unzulänglich sind, um das Gleichgewicht der Konstanz zu erreichen, genügt ein schwacher ventraler Strom, der an sich keine sichtbare Veränderung im Verhalten des Präparates herbeigeführt haben würde, um normales Konstanzniveau zu erreichen und zu behaupten. Das Gleiche hätte mit einer Last von 20 g direkt erreicht werden können. Wenn man den ventralen Strom unterbricht, ist der Tonusspiegel erniedrigt. Nunmehr wirken 20 g als „hohe" Last, die in aufeinanderfolgenden Wiederholungskurven mehr und mehr das Konstanzniveau herabdrückt. Bei dorsalem Strom steigt das Niveau; 20 g werden auf gleichem Niveau getragen als 10 g.

II. *Chemischer Einfluss auf den Aktivitätszustand beider Zentren nach unsern neuen Gesichtspunkten.*

1. *Ein Kochsalzkristall kommt auf die ventrale Seite der Ganglien.*

a. *Untersucht werden die* **Wiederholungskurven**. Die erste, steile Strecke der Dehnungskurven ist nunmehr steiler und länger, als im Vorversuch bei „normalem" Zustand

der Ganglien. Das Niveau der Pedalkonstanz wird erniedrigt. Man kann beim gleichen Präparat das Kochsalz ventral wegnemen und es, nach Drehung der Ganglienmasse um 180°, dorsal auflegen. Dann wird die erste Strecke der folgenden Wiederholungskurven weniger steil und die Pedalkonstanz kommt auf ein höheres Niveau.

b. **Die „ursprüngliche" Kurve.** Ein Kochsalzkristall kommt vor Aufnahme der ersten („ursprünglichen") Dehnungskurve auf die ventrale Seite der Pedalganglien. Die erste steile Dehnungsstrecke tritt nunmehr in 100 % der Fälle auf. Die Pedalkonstanz kommt um 13.8 % der Normallänge (der sogenannten Spannungslänge) niedriger zu liegen. Um dieser Resultate sicher zu sein, darf man höchstens 5 Minuten nach Auflegen des Kristalles mit der Aufnahme der Dehnungskurve warten. Wenn man 15 Minuten gewartet hat, so tritt eine entgegengesetzte Abänderung auf. Die steile Strecke ist nur in 25 % der Fälle nachzuweisen (gegenüber 61 % normal) und das Niveau der Pedalkonstanz steigt und zwar um 15.4 % der Muskellänge, die normaler Pedalkonstanz entsprochen haben würde. Offenbar besitzen die Pedalganglien ein ventrales Hemmungs- und ein dorsales Erregungszentrum für den Tonus; nach längerer Zeit erreicht das diffundierende Kochsalz, von der ventralen Seite kommend, auch das dorsale Zentrum, welches, wenn einmal seine Schwelle überschritten worden ist, die Erscheinungen beherrscht.

2. *Kokain. Versuche an Wiederholungskurven.*

Kokain dringt offenbar schnell in das Gangliengewebe ein, sodass die Dämpfungserscheinungen, die es nach Vergiftung verursacht, schnell auftreten. Für unsere Versuche aber hat dieser Umstand die nachteilige Folge, dass die Resultate schon nach kurzer Zeit ihre Eindeutigkeit verlieren, da das Kokain seine Wirkung dann nicht mehr auf das untersuchte Zentrum (hier also das ventrale Hemmungszentrum) beschränkt. Es tritt dann nämlich zugleich Dämpfung des Erregungszentrums auf, deren Wirkung den Effekt der Vergiftung des Hemmungszentrums bedeckt: Die erste Wiederholungskurve nach Auftragen des Kokains kommt in der Regel zu früh, um überhaupt Wirkung zu zeigen, die

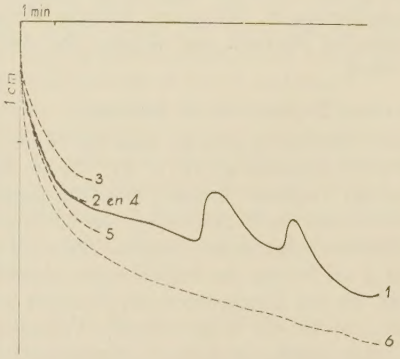

Abb. 2. Wiederholungskurven von einem pedaltonischen Präparat, Ganglien um 180° gedreht (um die Längsachse). Versuche mit Kokain auf der ventralen Seite der Ganglien. Dehnungskurven mit 20 g Last; 1 und 6 von 10 Minuten Dauer; die übrigen von 2 Minuten Dauer. 1: Ohne Kokain, Normalkurve; nach zwei Aufnahmen wird Kokain appliziert. 2: 6 Minuten nach Applikation (zu kurze Einwirkungsdauer, noch kein Effekt). 3: nach 16 Minuten: Dämpfung des ventralen lösenden Zentrums maximal. Sodann erreicht das diffundierende Kokain das dorsale Erregungszentrum mehr und mehr. 4: nach 21 Minuten Gleichgewicht zwischen Hemmungs- und Erregungsdämpfung. 5: nach 31 und 6: nach 36 Minuten (zunehmende Erregungsdämpfung): Erniedrigung des Tonusspiegels und des Niveau.

zweite aber, bei der üblichen Kurvendauer von 10 Minuten schon zu spät für reine Dämpfung der Hemmung.

Daher mussten wir die Zeitdauer der Aufnahme der einzelnen Wiederholungskurven auf einige Minuten beschränken. Die erste steile Kurvenstrecke geht nun, durch Dämpfung der Hemmung, verloren; in aufeinanderfolgenden Wiederholungskurven wird diese Dehnungsstrecke weniger steil, bis etwa 16 Minuten nach der Vergiftung (Abb. 2). Dann wird die Strecke wieder steiler und länger (Dämpfung des erregenden Zentrums); sehr deutlicher Unterschied mit Kurve 3 ist nach etwa 31 Minuten (Kurve 6) festzustellen. In der kurzen Zeit, welche diese Kurvenaufnahmen dauern dürfen, wird keine Konstanz erreicht, sodass wir über den Einfluss rein ventraler Dämpfung auf das Konstanzniveau nichts aussagen können. Schon bei einer Dehnung, die 10 Minuten dauert, fanden wir, vor allem bei nicht zu geringer Last, Niveauverlust, welcher ohne Kokainvergiftung nicht aufgetreten sein würde; Niveauerhöhung wurde aber niemals wahrgenommen. Da bald danach auch, wie gesagt, Verlängerung der steilen Strecke auftritt, schliessen wir auf Dämpfung des erregenden Zentrums, die inzwischen weiter vorgeschritten ist. Die früher oft störend empfundene Zweideutigkeit der Resultate bei (dorsaler!) Ganglienkokainisierung, ist daher offenbar als ein Streit zwischen beiden Zentren, je nach ihrem Vergiftungsgrad, aufzufassen.

III. *Tonuslösung durch Reizung mit Induktionsströmen.*

Auf Grund unsrer Ergebnisse haben wir ältere Versuche wieder aufgenommen, um die Frage zu entscheiden, ob man Tonuslösung auch durch Reizung hervorrufen kann. Bislang war uns das nicht gelungen. Nur der Fall des Tonusspiegels, den wir bei Auflegen eines Kochsalzkristalles dorsal auf die Pedalganglien beobachtet hatten, wies auf die Möglichkeit eines positiven Ergebnisses direkter Ganglienreizung hin.

1. *Reizung der Pedalganglien durch Einzelöffnungsschläge.*

a. Ventrale Reizung (Abb. 3). Schwellenreize (R. A. 30 cm) erzielen geringfügigen Tonusfall, der mit der Reizstärke zunimmt (bis R. A. 26 cm). Bei R. A. 28 cm wird die Schwelle des erregenden Zentrums überschritten. Bei R. A. 22 cm trat reine Kontraktion auf. Ein neutrales Gebiet der Reizgrösse, bei welcher also keinerlei Reaktion auftritt, war nicht festzustellen. Eventuelle Muskelverkürzungen kommen stets vor der Tonuslösung.

b. Dorsale Reizung (Kathode rostro-dorsal). Die schwächsten Reaktionen (R. A. 55—53 cm) bestehen aus Spiegelerniedrigung (während bei ventralen Reizen lediglich Niveauerniedrigung, d.h. kurzes Absinken der Kurve, auftritt). Daher verhindert schwache dorsale Reizung die Behauptung einer Konstanz. Bei zunehmender Reizstärke (R. A. 48—40) fanden wir ein neutrales Gebiet, wobei also keinerlei Reaktion auftrat. Bei R. A. 35 und 34 cm fand sich geringe Tonuserhöhung; Reizung bei R. A. 25 cm erzeugte Kontraktion.

2. *Reflexreizung (Einzelöffnungsschläge) eines der Nn. palliales.*

Auch hier fanden wir bei schwacher Reizung Tonuslösung, bei starken Reizen nur Kontraktion, während auch hier zwischen beiden Gebieten eine neutrale Zone liegt, in welcher offenbar die Wirkungen beider Zentren miteinander im Gleichgewicht sind.

3. *Reizung der Pedalnerven mit Einzelöffnungsschlägen und Nervendurchschneidung.*

a. Elektrische Reizung der **caudalen Nerven** mittels untergeschobener Elektroden; die Kathode liegt myopolar; zwischen Anode und Pedalganglien sind die Nervenstränge durch Erhitzen zerstört, um Reflexwirkung auszuschliessen. Auch bei dieser Technik ergab sich ein Unterschied zwischen schwacher und starker Reizung. Schwache Reize verursachen Fall der Kurve (nach langer Latenz). Stärkere Reize verursachen erst Kontraktion, sodann Tonusfall. Zuweilen heben beide Wirkungen einander auf. Starke Reize haben Kontraktion zur Folge, hiernach bleibt der Tonusspiegel erhöht.

b. Um **alle Pedalnerven** reizen zu können, wurden sie in Strängen zusammengebunden; die Knoten lagen nahe an der Ganglienmasse. Nachdem die Elektroden auf den Strängen befestigt waren, wurden die Stränge von den Pedalganglien getrennt. Erst wurden die Versuche mit elektrischer Reizung ausgeführt; dann reizten wir die Nerven mechanisch.

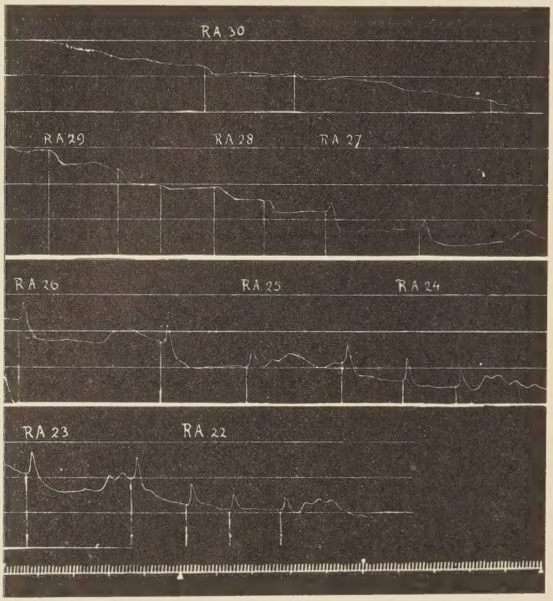

Abb. 3. Ursprüngliche Kurve eines Schneckenfusses mit Ganglienmasse, die 180° um die Längsachse gedreht ist. Reizung der Ganglien mit Einzelöffnungsschlägen, Kathode auf der ventralen Seite, Anode indifferent. R. A. bedeutet Reizstärke in cm Rollenabstand. Liminale Reize lösen nur Niveauverlust aus (R. A. 30 und 29); stärkere Reize ergeben mehr Niveauverminderung, denen eine Kontraktion vorausgeht (R. A. 28 bis 23). Starke Reize haben nur Kontraktion zur Folge (R. A. 22). Die vertikalen Linien geben den Augenblick der Reizung an.

α. Mechanische Reize (Durchschneidung der Nervenbündel). Diese Methode, bei der Veränderung der Reizstärke ausgeschlossen ist, ergab bei Durchschneidung rostraler Stränge nichts oder Erschlaffung, bei Durchschneidung caudaler Stränge Verkürzung oder nichts.

β. Elektrische Einzelreihe bestätigen die Versuche mit Durchschneidung (Abb. 4): Schwächere Reize wirken lösend, wobei die Tonuslösung mit der Reizstärke abnimmt. Stärkere Reize haben Verkürzung mit Erhöhung des Tonusniveau zur Folge. Auch hier zeigten die rostralen Stränge überwiegend lösende Wirkung. Offenbar laufen in den Pedalnerven spezifisch hemmende (neben erregenden) Fasern, die in den rostralen Strängen überwiegen, und in allen Strängen niedrigere Reizschwelle haben als die erregenden Fasern.

C. Zusammenfassung.

1. Um den Aktivitätszustand der Zentren streng lokalisiert zu erhöhen und zu erniedrigen werden die Pedalganglien von Helix pomatia während der Dehnung mit schwachen konstanten Strömen durchströmt. Hierbei liegt in einer Versuchsreihe die Kathode dorsal

und die Anode ventral, in andern Versuchen umgekehrt. Dorsaler Katelektrotonus erhöht das Niveau der Pedalkonstanz, ventraler Katelektrotonus erniedrigt dieses Niveau.

2. Kochsalzkristalle, ventral aufgelegt, verstärken die Tonuslösung: die erste steile Strecke der Dehnungskurve tritt nunmehr in 100 % der Kurven auf; sie ist länger als in Versuchen ohne Aktivierung des ventralen Teiles der Ganglien. Das Niveau der Pedalkonstanz wird erniedrigt. Auf die Dauer steigt das Niveau, da zweifellos das Kochsalz bis in den dorsalen Teil der Ganglien durchdringt.

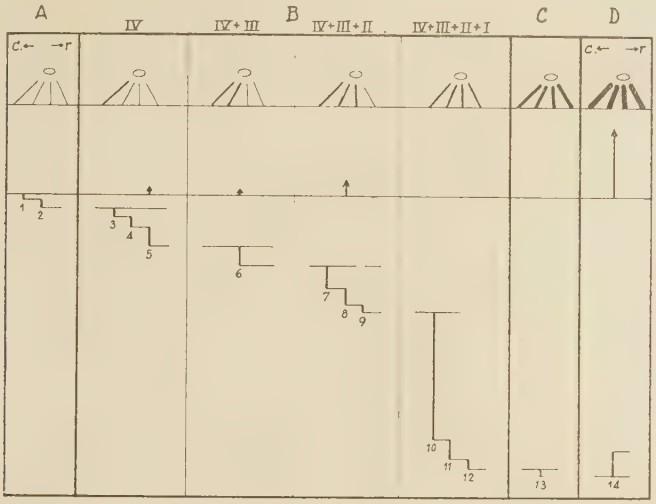

Abb. 4. Schema der Effekte einer Reizung der Nn. pedales eines Schneckenfusses ohne Ganglien; Einzelöffnungsschläge. *Erste horizontale Reihe:* die vier (IV bis I) von den Pedalganglien getrennten Nervenstränge; c = caudale, r = rostrale Richtung. Auf jedem Strang wird eine Anode und eine Kathode (letztere myopolär) befestigt, welche verbunden sind mit je einer Sekundärrolle. In allen vier Sekundärrollen werden die Ströme gleichzeitig durch eine einzige Primärrolle induziert. In der ersten vertikalen Kolonne sind alle Sekundärrollen um einen gleichen grösseren Abstand von der gemeinsamen primären entfernt (Reizstärke A). Dann wird die Rolle des Stranges IV näher zur primären geschoben, dieser Strang also stärker gereizt (Stärke B); später folgt Rolle III, u.s.w. Nachdem alle vier Stränge mit Stärke B gereizt worden sind, werden alle vier sekundäre Rollen noch mehr der primären Rolle genähert: Reizstärke C. Die Reizstärke wird sodann für alle 4 Stränge noch einmal verstärkt (Stärke D). Die Stränge sind nach Massgabe der zunehmenden Reizstärke dicker gezeichnet. *Zweite horizontale Reihe:* Die Pfeile bedeuten Kontraktionsausschläge, als auf stillstehendem Kymographion aufgezeichnet. *Dritte horizontale Reihe:* Niveauverlust oder -erhöhung, auf stillstehendem Kymographion.

Ergebnisse: Reizstärke A löst nur Erschlaffung aus. Stärke B ruft in den Strängen IV, III und II Tonusverlust, bisweilen (Reizung 5, 6 und 8) vorher auch Verkürzung hervor; sobald Strang I hinzukommt, erfolgt ausgiebige Erschlaffung (10), keine Kontraktion. Erst mit sehr starken Reizen (D) tritt Kontraktion mit Tonussteigerung auf (14).

3. Kokain, ventral appliziert, bringt die steile Strecke zum Verschwinden und das Niveau der Pedalkonstanz wird erniedrigt. Später tritt die steile Strecke wieder auf und die Pedalkonstanz verschwindet. Zweifellos hat das diffundierende Kokain inzwischen den dorsalen Teil der Ganglien erreicht.

4. Reizung der ventralen Seite der Ganglien mit liminalen Einzelöffnungsschlägen verursacht Verminderung des Tonusniveau; bei stärkeren Reize tritt erst Kontraktion, sodann grössere Verminderung des Tonusniveau auf. Bei starken Reizen treten lediglich Einzelverkürzungen, ohne Einfluss auf das Niveau der Konstanz auf.

5. Schwache Reizung der dorsalen Seite der Ganglien ruft Verminderung des tonischen Widerstandes im Laufe der Dehnung hervor. Stärkere Reizung bleibt ohne Effekt, starke Reizung hat Einzelverkürzungen, ohne Einfluss auf den Tonus, zur Folge. Reizung eines Pallialnerven ergibt Resultate wie dorsale Ganglienreizung.

6. Bei Reizung der Pedalnerven (ohne Pedalganglien) zeigt sich, dass die Pedalstränge hemmende und erregende Fasern führen. Die hemmenden Fasern finden sich überwiegend in den rostralen Strängen. Sie haben in allen Strängen eine niedrigere Schwelle als die erregenden Fasern.

7. *Allgemeine Schlüsse aus diesen Ergebnissen:* Die Resultate zwingen uns, das Vorhandensein *eines dorsalen erregenden und eines ventralen hemmenden Tonuszentrums* in den Pedalganglien anzunehmen. Im Gegensatz zu unserer früheren Auffassung werden die pedaltonische Erscheinungen (Spiegelerniedrigung und Konstanzerzeugung) quantitativ durch das Verhältnis des Aktivitätszustandes beider Zentren bestimmt. Offenbar erfolgt jede Form von *zentraler Regulierung durch Impulse,* im Falle von Tonuserniedrigung durch *spezifische Hemmungsfasern* in den Pedalnerven, vor allem in den rostralen, *welche Fasern eine niedrigere Reizschwelle haben als die erregende Fasern.*

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Hydrodynamics. — *On the influence of the concentration of a suspension upon the sedimentation velocity* (in particular for a suspension of spherical particles) *). By J. M. BURGERS. (Mededeeling No. 42 uit het Laboratorium voor Aero- en Hydrodynamica der Technische Hoogeschool te Delft.)

(Communicated at the meeting of November 29, 1941.)

8. In the preceding part of this paper a general picture has been given of the problem of the sedimentation velocity of small particles in a viscous liquid, and those features have been pointed out which require a more thoroughgoing investigation. The difficulties inherent in the problem are mainly due to the circumstance that according to STOKES's equations the velocities produced in an unlimited field by a moving particle A decrease proportionally to the inverse *first* power of the distance. The influence exerted upon a neighbouring particle B in such a field appears to be always of the same sign (the particle B is dragged along in the direction of the motion of A), and is of the order of magnitude a/r, a being the radius of a particle and r the distance between A and B. In an assembly containing a great number of particles the contributions to be derived from more and more distant groups consequently do not form a converging series, and the resulting effect will be dependent upon the extent and the shape of the assembly, as well as upon the conditions imposed by the properties of the surrounding field.

When the suspension is enclosed in a vessel, a return flow somewhere must occur, as was mentioned before in **3.**; in this case, along with the positive effect between neighbouring particles, there will be a negative effect between particles at distances from each other comparable with the dimensions of the vessel. In consequence of the fact that these two effects nearly balance each other, the resulting average effect (which appears to be of negative sign) becomes of a much higher order of smallness, *viz.* of the order $n\,s$, where n is the number of particles in the suspension per unit volume, and s is the volume of a particle. When the mean distance between two particles is put equal to r_{mean}, which quantity will be of the order $n^{-1/3}$, we can also say that the resulting average effect is of the order a^3/r^3_{mean}.

It may be surmised that the precise numerical magnitude of this resulting average effect should be rather sensitive to circumstances which may affect the balancing between the "positive" and the "negative" regions of the field. In particular it might be supposed that the form of the vessel — which is the determining factor in the calculation of the function χ in eq. (12) — greatly may affect this balancing. In the developments of sections **5.**—**7.** this possibility presented itself in the result that the equation derived for the mean value of Φ_m contained terms depending upon the value of $\partial\Phi/\partial n$ along the walls of the vessel, for which a general expression could not be found.

The possibility that the sedimentation velocity should be dependent upon the shape of the vessel nevertheless does not appear to be readily acceptable [5]). In the discussion of the paper at the meeting of the Academy of October 25 Professor VENING MEINESZ raised the question whether it would be possible to solve the problem for a suspension

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*) Continued from these Proceedings **44**, 1941, p. 1051. — The reader will have observed that an error of print has crept into the formula for r, given at p. 1048, line 6 from above; this formula must read:

$$r = \{(x-\xi)^2 + (y-\eta)^2 + (z-\zeta)^2\}^{1/2}.$$

[5]) It should be stated that in all cases to be considered the suspension is assumed to be homogeneous; *i.e.* we suppose that the number of particles per unit volume (n) everywhere in the field has the same value.

extending indefinitely in all directions, provided the boundary condition which in the case of a suspension enclosed in a vessel is imposed by the impermeability of the walls were replaced by an equivalent condition of another type. This cannot be done, however, so long as the solution for the infinite field as given by STOKES is taken as a basis. In a private discussion with Professor BIEZENO, to which I owe much in clearing up the difficulty encountered, the latter proposed to replace STOKES' solution by a different one, in which a return flow in the more distant parts of the field is automatically combined with the ordinary STOKES' flow in the immediate neighbourhood of the particle [6]).

It appears that this actually can be done. Although the procedure at first sight may seem to be somewhat artificial, it will be shown that it can be used in constructing a solution of the problem, which will enable us to treat the various questions mentioned before — the one concerning the correction for the hydrostatic pressure, that for the effective viscosity, and that for the field of flow — from a unified point of view. The procedure to be followed at the same time makes it possible to obtain an estimate of the influence of a preferential distribution of the distances of the particles in the neighbourhood of a given one, such as may appear *e.g.* when repulsive forces are acting between the particles [7]).

9. The solution to be used represents the field of flow produced by a concentrated force of magnitude F acting at the origin, combined with a "diffuse" field of force given by:

$$X = -(\varkappa^3 F/8\pi)\, e^{-\varkappa r} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (25)$$

Here $\varkappa$ is a small quantity, of such nature that $1/\varkappa$ represents a distance large in comparison with the average distance between the particles. All forces are in the direction of the x-axis. The integral of X taken over the whole field is equal to $-F$, so that the force system is an *equilibrium system.*

Introducing the function:

$$\Psi = + \frac{F}{8\pi\varkappa\eta}\left[e^{-\varkappa r} + \frac{4\, e^{-\varkappa r}}{\varkappa r} - \frac{4}{\varkappa r}\right] \quad . \quad . \quad . \quad . \quad . \quad (26)$$

which satisfies the relation:

$$\eta \triangle \triangle \Psi = -X \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (26a)$$

(η being the true viscosity of the pure liquid) the field of flow can be described by the equations:

$$u_1 = \triangle \Psi - \partial^2 \Psi/\partial x^2; \quad v_1 = -\partial^2 \Psi/\partial x\, \partial y; \quad w_1 = -\partial^2 \Psi/\partial x\, \partial z \quad . \quad . \quad (27)$$

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[6]) A field of flow differing from that determined by STOKES's formulae, is also obtained when OSEEN's theory is applied (compare *e.g.* J. M. BURGERS, these Proceedings **23**, 1921, p. 1087 and fig. 11 at p. 1091). The distribution of the current given by OSEEN's theory, however, is dependent upon the magnitude of the REYNOLDS number $Re = 2\varrho u_0 a/\eta$ (u_0 being the velocity of the particle); moreover its general character makes it appear less promising than the type of flow discussed below in section **9**.

[7]) As was pointed out by Professor KRUYT at the meeting of the Academy, the presence of repulsive forces must be considered as an essential feature of the majority of stable suspensions. Hence the assumption provisionally made in the preceding part of the paper, that the particle B may take in all positions relatively to A with equal probability, in many cases will not be applicable.

When the pressure p is taken equal to $-\eta\partial(\triangle\Psi)/\partial x$, then in consequence of the relation (26a) this field satisfies the equations:

$$\eta\triangle u = \partial p/\partial x - X; \quad \eta\triangle v = \partial p/\partial y; \quad \eta\triangle w = \partial p/\partial z,$$

together with the equation of continuity.

In order to obtain a field which at the same time will satisfy the boundary conditions at the surface of a spherical particle of radius a, we add a second field of flow, described by the equations:

$$u_{II} = \partial^2\varphi/\partial x^2; \quad v_{II} = \partial^2\varphi/\partial x\,\partial y; \quad w_{II} = \partial^2\varphi/\partial x\,\partial z \quad . \quad . \quad (28a)$$

where:

$$\varphi = -Fa^2/24\pi\eta r \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (28b)$$

As $\triangle\varphi = 0$, this field satisfies the equations of motion with $p = 0$, $X = 0$ [8]).

For the resulting field we find:

$$u = u_I + u_{II} = \frac{F\varkappa}{8\pi\eta}\left[e^{-\sigma}\left\{\left(1+\frac{3}{\sigma}+\frac{4}{\sigma^2}+\frac{4}{\sigma^3}\right) - \right.\right. \\ \left.\left. -\frac{x^2}{r^2}\left(1+\frac{5}{\sigma}+\frac{12}{\sigma^2}+\frac{12}{\sigma^3}\right)\right\} - \left(1-\frac{3x^2}{r^2}\right)\frac{4}{\sigma^3}\right] + \frac{F}{8\pi\eta}\left(\frac{a^2}{3r^3}-\frac{a^2x^2}{r^5}\right) \quad (29)$$

where for shortness has been written: $\sigma = \varkappa r$.

So long as $\varkappa r$ is small in comparison with unity, we may develop the exponential function; we then find:

$$\Psi = \text{const.} + \frac{Fr}{8\pi\eta} - \frac{F\varkappa r^2}{48\pi\eta} \quad . \quad . \quad . \quad . \quad . \quad . \quad (30a)$$

$$u = \frac{F}{8\pi\eta}\left\{\frac{1}{r}+\frac{x^2}{r^3}+\frac{a^2}{3r^3}-\frac{a^2x^2}{r^5}-\frac{2\varkappa}{3}\ldots\right\} \quad . \quad . \quad . \quad (30b)$$

where the terms not written out are of the order $\varkappa^3r^2$ and vanish for $r = 0$. At the surface of the spherical particle, where $r = a$, the velocity becomes: $u_0 = (F/6\pi\eta a)\,.\,(1-\frac{1}{2}\varkappa a\ldots)$. In view of the condition imposed upon $\varkappa$ the term $\frac{1}{2}\varkappa a$ is so small in comparison with unity that it wholly can be neglected; we therefore come back to the velocity which is given by the ordinary STOKES' formula, *viz.*:

$$u_0 = F/6\pi\eta a. \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (30c)$$

On the other hand for values of r so large that $\varkappa r$ is large in comparison with unity, the exponential function decreases to zero, and there remains:

$$\Psi = -F/2\pi\varkappa^2\eta r \ (\text{with}: \triangle\Psi = 0) \quad . \quad . \quad . \quad . \quad . \quad . \quad (31a)$$

$$u = -\frac{F}{8\pi\eta}\left(\frac{4}{\varkappa^2}-\frac{a^2}{3}\right)\left(\frac{1}{r^3}-\frac{3x^2}{r^5}\right). \quad . \quad . \quad . \quad . \quad . \quad (31b)$$

It will be observed that the mean value of u taken over a spherical surface ($r = \text{constant}$) becomes zero in this case.

It may be mentioned that a current function ψ can be constructed, so that:

$$u = \frac{1}{2\pi\omega}\frac{\partial\psi}{\partial\omega}; \quad v_\omega = -\frac{1}{2\pi\omega}\frac{\partial\psi}{\partial x} \quad . \quad . \quad . \quad . \quad . \quad . \quad (32)$$

---

[8]) It can be shown that the field of flow described by eqs. (28a), (28b) may be considered as being produced by a multiplet consisting of six equal forces of magnitude f, respectively acting at the points $+\varepsilon, 0, 0; -\varepsilon, 0, 0; 0, +\varepsilon, 0; 0, -\varepsilon, 0; 0, 0, +\varepsilon; 0, 0, -\varepsilon$; combined with a force of magnitude $-6f$, acting at the origin, provided $2f\varepsilon^2 = Fa^2/3$.

The equilibrium of the whole system thus is not disturbed.

where: $\omega = (y^2 + z^2)^{1/2}$; $v_\omega = (v^2 + w^2)^{1/2}$. This function is given by:

$$\psi = -\frac{F\,\omega^2}{4\,\eta\,r}\left[e^{-\tau}\left(1+\frac{4}{\sigma}+\frac{4}{\sigma^2}\right)-\frac{4}{\sigma^2}+\frac{a^2}{3\,r^2}\right] \quad . \quad . \quad . \quad (33)$$

This expression becomes zero for $\omega = 0$ and for $\omega = \infty$. Hence, provided x is different from zero, the integral $\iint dy\,dz\,u$, extended over an unlimited plane $x =$ constant, is zero. — When r is equal to a, the expression (33), after development of the exponential function, takes the value: $\psi = (F\,\omega^2/6\,\eta\,a)\ (1 - \frac{1}{2}\,\varkappa\,a) = \pi\,\omega^2\,u_0$, from which it follows that: $\iint dy\,dz\,u = -\pi\,\omega^2\,u_0$, when the integral is extended over that part of a plane $x =$ constant (with $|x| < a$), which lies outside of the spherical surface $r = a$. Hence:

$$\int\int\int dx\,dy\,dz\,u = -(4\pi\,a^3/3)\,u_0 \quad . \quad . \quad . \quad . \quad . \quad (34)$$

where the integral is extended over the whole of space outside of the spherical surface $r = a$.

10. We now are in a position to build up a description of the field of flow which is adapted to our purpose.

The suspension consists of a liquid of density ϱ, which is acted upon by a force $\varrho\,g$ per unit volume, and in which are embedded n particles per unit volume, every one of which is acted upon by a force $W = s\,\varrho_P g$ (s again being the volume and ϱ_P being the density of a particle). All forces act in the same direction, which is assumed to be the direction of the x-axis. The total system of forces can be analysed into the following components:

a) a continuous field of force with the intensity $\varrho\,g + n\,F$ per unit volume (where F will be determined below), acting through the whole space, *i.e.* both through the space taken in by the liquid and through that taken in by the particles;

b) a set of "equilibrium systems" of the type considered in **9.**, each system having its centre at the centre of a particle, so that there are n systems per unit volume of the field.

It is not difficult to prove from eq. (25) that in an infinitely extended field the resultant effect of the "diffuse" fields of the equilibrium systems is equivalent to a continuous force of magnitude $-n\,F$ per unit volume, acting throughout the whole field [9]). Combining this with the field mentioned under *a*), we come back upon the force $\varrho\,g$ per unit volume acting upon the liquid, and a force $s\,\varrho\,g$ acting upon every particle. The total force acting upon a particle thus will have the value $F + s\,\varrho\,g$. As this must be equal to W, we obtain the equation:

$$F = W - s\,\varrho\,g = s\,(\varrho_P - \varrho)\,g \quad . \quad . \quad . \quad . \quad . \quad . \quad (35)$$

The field of force considered under *a*) does not give rise to any flow. It is kept in equilibrium by a pressure gradient of the constant magnitude [10]):

$$\partial p_m/\partial x = \varrho\,g + n\,F = \varrho\,g + n\,s\,(\varrho_P - \varrho)\,g\,. \quad . \quad . \quad . \quad (36)$$

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[9]) This result will be more and more accurate when $\varkappa$ is so small that the irregularities in the spatial distribution of the particles are averaged out over distances of the order $1/\varkappa$.

[10]) This pressure gradient may be assumed to act through the interior of a particle. It is kept in equilibrium there by a force of magnitude $s.(\partial p_m/\partial x)$ in the direction of the x-axis. If we add to this the force $-s\,n\,F$, which is necessary to balance the pressure distribution corresponding to the continuous force $-n\,F$ that was obtained as the resultant of the "diffuse" fields, and further the force F itself acting upon a particle, we obtain the total amount:

$$F - s\,n\,F + s.(\partial p_m/\partial x) = W$$

by (35) and (36).

As the system of forces considered under *b*) is an equilibrium system, which, although it may give rise to local pressures, will not produce a constant average pressure gradient throughout the whole field, the expression (36) at the same time represents the *mean hydrostatic pressure gradient* in the field. This corresponds to what was mentioned in connection with eq. (2) of the preceding part of this paper.

On the other hand, every one of the "equilibrium systems" will produce a field of flow as described by the equations of section **9**. The force F determining the intensity of a single system appears to be given by eq. (35), *i.e.* by eq. (1) of the preceding part of this paper with $\varrho_S = \varrho$, that is, with ϱ_S equal to *the density of the pure liquid*. Hence the correction indicated by eq. (3) must *not* be applied when the calculations are performed according to the scheme developed here.

There now is no longer any danger that the contributions to the velocity at an arbitrary point of the field, to be derived from the infinite number of particles, will form a divergent series. This follows from the result embodied in eq. (34), and will appear also from the calculations to be developed later on in section **17**.

11. We now proceed to the investigation of the influence exerted by a particle A upon another particle B, and of B's reaction upon A. It is convenient again to assume provisionally that only A is acted upon by the gravitational force, whereas B is supposed to have the same density as the liquid, so that it is carried along by the motion of the latter.

If the components of the velocity of the flow, produced by the motion of A, are denoted by u, v, w, it follows from the considerations of section **4**. that the components of the velocity taken up by B will be:

$$u_m = u + \tfrac{1}{6} a^2 \triangle u; \quad v_m = v + \tfrac{1}{6} a^2 \triangle v; \quad w_m = w + \tfrac{1}{6} a^2 \triangle w. \quad (37)$$

u, v, w here representing the values of these quantities at the centre of B.

The particle B at the same time will take up a rotation, the components of the angular velocity having the magnitude:

$$\omega_x = \tfrac{1}{2} (\partial w/\partial y - \partial v/\partial z), \text{ etc.} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (38)$$

where the values of the derivatives likewise must be taken at the centre of B.

When these motions (translation and rotation) are given to B, there will not yet be equality of the velocity of the liquid on the one hand, and the velocity of the points of the surface of B on the other hand. The difference between the velocity components of the liquid and those of the surface of B can be expressed by means of the formulae:

$$\left.\begin{aligned} u^* &= x' D_{11} + y' D_{12} + z' D_{13} + \text{terms of the 2nd, 3rd, ... degree in } x', y', z' \quad . \quad . \quad . \quad . \quad . \\ v^* &= x' D_{21} + y' D_{22} + z' D_{23} + \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \\ w^* &= x' D_{31} + y' D_{32} + z' D_{33} + \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \end{aligned}\right\} \quad (39)$$

Here x', y', z' are the coordinates of a point of the surface of B, with the centre of B as origin; further:

$$D_{11} = \partial u/\partial x; \quad D_{12} = D_{21} = \tfrac{1}{2} (\partial v/\partial x + \partial u/\partial y), \text{ etc.} \quad . \quad . \quad . \quad (40)$$

The mean values of the terms of the second degree in the expressions (39) for u^*, v^*, w^*, taken over the surface of the sphere, are zero when the values (37) are given to the components of the translational velocity of B.

It is possible to get rid of the terms of the first degree in the expressions (39) by assuming in B to be present certain doublets, combined with some multiplets of higher order, by means of which an appropriate system of additional velocities is called forth in

the liquid. In order to do this most conveniently we introduce a new system of coordinates x'', y'', z'', orientated in such a way that in this system $D_{12} = D_{13} = D_{23} = 0$. We begin by taking $z' = 0$; then the x', y'-plane becomes a meridian plane, and $D_{13} = D_{23} = 0$.

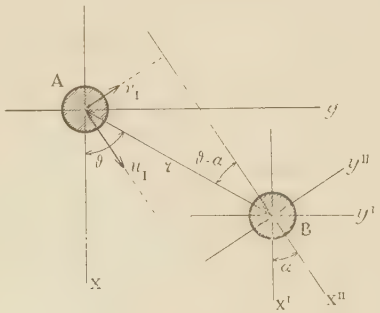

Fig. 1.

In the meridian plane we introduce axes x'', y'' (compare fig. 1); the angle α between the x''-axis and the x'-axis being determined by: $tg\ 2\alpha = 2D_{12}/(D_{11} - D_{22})$. With $z'' = z'$ the linear terms of (39) can be transformed into:

$$u^* = x'' D_1; \quad v^* = y'' D_2; \quad w^* = z'' D_3 \quad . \quad . \quad . \quad . \quad (41)$$

(all quantities now referring to the new axes), where:

$$\left.\begin{aligned} D_1 &= D_{11} \cos^2\alpha + D_{22} \sin^2\alpha + 2 D_{12} \sin\alpha \cos\alpha \\ D_2 &= D_{11} \sin^2\alpha + D_{22} \cos^2\alpha - 2 D_{12} \sin\alpha \cos\alpha \\ D_3 &= D_{33} \end{aligned}\right\} \quad . \quad . \quad . \quad (42)$$

It will be seen that: $D_1 + D_2 + D_3 = D_{11} + D_{22} + D_{33} = 0$.

12. The field of flow produced by a doublet is given by eqs. (9.3) of the "Second Report on Viscosity and Plasticity" (p. 133). We introduce 3 doublets, having the following orientations: M_1 parallel to the x''-axis; M_2 parallel to the y''-axis; M_3 parallel to the z''-axis. It can be deduced from the equations mentioned that the resulting field vanishes when $M_1 = M_2 = M_3$. Consequently without loss of generality we can take: $M_1 + M_2 + M_3 = 0$, so that $M_1 = -(M_2 + M_3)$.

The expressions for the components of the velocity produced by these doublets can then be brought into the form:

$$u_I = -(3x''/8\pi\eta r''^3) Q_1; \quad v_I = -(3y''/8\pi\eta r''^3) Q_1; \quad w_I = -(3z''/8\pi\eta r''^3) Q_1 \quad (43)$$

where $r'' = (x''^2 + y''^2 + z''^2)^{1/2}$, and:

$$Q_1 = M_2 \left(1 - \frac{2y''^2 + z''^2}{r''^2}\right) + M_3 \left(1 - \frac{y''^2 + 2z''^2}{r''^2}\right).$$

The field produced by the doublets M_1, M_2, M_3 still must be combined with a field produced by two multiplets of higher order, presenting the velocity components:

$$u_{II} = x'' Q_2; \quad v_{II} = y'' (Q_2 + 6B/r''^5); \quad w_{II} = z'' (Q_2 + 6C/r''^5) \quad (44)$$

where:
$$Q_2 = B \left(\frac{3}{r''^5} - \frac{15 y''^2}{r''^7}\right) + C \left(\frac{3}{r''^5} - \frac{15 z''^2}{r''^7}\right).$$

We now introduce the condition that the resulting field produced by these doublets and multiplets shall annul the quantities given by (41) at the surface of the spherical particle B. This requires:

$$u^* + u_I + u_{II} = 0, \text{ etc. for } r'' = a \quad . \quad . \quad . \quad . \quad . \quad . \quad (45)$$

The solution of these equations gives:

$$B = (2M_2 + M_3)\, a^2/40\pi\eta; \quad C = (M_2 + 2M_3)\, a^2/40\pi\eta \quad . \quad . \quad (46)$$

and:
$$M_1 = -2k\eta D_1; \quad M_2 = -2k\eta D_2; \quad M_3 = -2k\eta D_3 \quad . \quad (47)$$

with: [11])
$$k = 10\pi a^3/3 = 2{,}5\, s. \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (48)$$

13. The next step is to find the velocity which the particle A derives from the additional field considered in the preceding section. We may call this the "induced velocity" of A. For this purpose we calculate the velocity of this field at the centre of A; in order to make the expressions not too complicated, we shall give attention to the field produced by the doublets M_1, M_2, M_3 only.

Referring to fig. 1 we must put in eqs. (43): $r'' = r$; $x'' = -r\ cos\ (\vartheta - \alpha)$; $y'' = -r\ sin\ (\vartheta - \alpha)$; $z'' = 0$; after some reduction we obtain:

$$u_I = +(5a^3/2r^2)\, Q_3 \cos(\vartheta - \alpha); \quad v_I = +(5a^3/2r^2)\, Q_3\, sin\,(\vartheta - \alpha); \quad w_I = 0 \quad (49)$$

where:
$$Q_3 = D_1\, cos^2\,(\vartheta - \alpha) + D_2\, sin^2\,(\vartheta - \alpha).$$

Hence the component in the x-direction of the "induced velocity" becomes:

$$u_i = u_I \cos\alpha - v_I\, sin\ \alpha = (5a^3/2r^2)\, Q_3 \cos\vartheta \quad . \quad . \quad . \quad . \quad (50)$$

In order to obtain the values of α, D_1, D_2 to be substituted into this expression, we introduce a further simplification and restrict to the most important terms in the equations for u, v, w. For shortness we omit the factor $F/8\pi\eta$; the expressions to be used then take the forms:

$$u = 1/r + x^2/r^3; \quad v = xy/r^3; \quad w = xz/r^3 \quad . \quad . \quad . \quad . \quad (51)$$

from which:

$$\left.\begin{aligned} D_{11} &= x/r^3 - 3x^3/r^5; \quad D_{12} = -3x^2 y/r^5 \\ D_{22} &= x/r^3 - 3xy^2/r^5 \\ D_{33} &= x/r^3 \end{aligned}\right\} \quad . \quad . \quad . \quad . \quad (52)$$

and:
$$\alpha = \vartheta; \quad D_1 = -2x/r^3; \quad D_2 = D_3 = +x/r^3. \quad . \quad . \quad . \quad (53)$$

Equation (50) then gives:

$$u_i = -5a^3\, x^2/r^6 = -5a^3 \cos^2\vartheta/r^4. \quad . \quad . \quad . \quad . \quad (54)$$

The reader will have observed that in the deduction of the last expressions terms which are of higher orders with respect to a/r have been neglected. The incorporation of these terms would have yielded rather cumbersome formulae; besides, in that case it would have become necessary to introduce a compensational field likewise for the still remaining terms of the second degree in (39), and possibly also for further reactions between the

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[11]) When this result is combined with eq. (8) of the preceding part of this paper, we obtain a proof of EINSTEIN's formula for the specific increase of the viscosity of a suspension of spherical particles.

particles A and B and other particles. The expression (54) consequently will be valid only for not too large values of the ratio a/r. It may be noted on the other hand, as was stated already in footnote 4) to section 4., that the errors may remain small even when the particles should happen to be at a small distance from each other.

14. Summing up the results obtained, we have found that, under the influence of the field of flow produced by the falling particle A, the particle B is carried along with a velocity, the component in the direction of the x-axis of which is given by:

$$u_m = u + \tfrac{1}{6} a^2 \triangle u \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (37a)$$

u being the velocity at the centre of B; whereas the particle A, in consequence of the disturbance due to the presence of B, experiences an induced velocity u_i, as given by (54), which is of negative sign and thus evidently represents a retardation of the motion of A.

In making use of (37a) we must take account of the terms of the order $1/r$ and those of the order a^2/r^3. For two particles at a distance r from each other, which still is small in comparison with $1/\varkappa$, these terms are sufficient. Writing $x = r \cos\vartheta$, we then have:

$$u = (1 + \cos^2\vartheta)/r + a^2 (1 - 3\cos^2\vartheta)/3r^3,$$

and:

$$u_m = (1 + \cos^2\vartheta)/r + 2a^2 (1 - 3\cos^2\vartheta)/3r^3 \quad . \quad . \quad . \quad (37b)$$

In (54) the approximation has been restricted to terms of the order a^3/r^4.

The expressions for u_m and u_i remain the same when the positions of the particles are interchanged; as they depend upon $\cos^2\vartheta$. Hence when now we consider two particles both of which experience the action of the gravitational force, they will acquire the same resultant velocity [12]):

$$u_{res} = \frac{F}{8\pi\eta}\left(\frac{4}{3a} + u_m + u_i\right) = \frac{F}{8\pi\eta}\left\{\frac{4}{3a} + \frac{1+\cos^2\vartheta}{r} + \frac{2a^2(1-3\cos^2\vartheta)}{3r^3} - \frac{5a^3\cos^2\vartheta}{r^4}\right\},$$

where the factor $F/8\pi\eta$ has been restored, and where terms of higher orders have been neglected [13]).

The investigation of the influence which a particle experiences from the surrounding ones in a suspension of unlimited extent will be given in a subsequent part of this paper.

(To be continued.)

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[12]) The two particles will also acquire velocities in directions perpendicular to the x-axis. It would appear that these velocities likewise are the same for both particles, so that their distance and their positions relatively to each other are not changed. Compare: C. W. OSEEN, Hydrodynamik (Leipzig 1927), p. 157.

[13]) In the expression between the brackets there would also appear the term $-2\varkappa/3$, and terms of the order $\varkappa^3 a^2$, etc. As mentioned already before, these certainly can be neglected, as it has been assumed that $\varkappa r \ll 1$, so that $\varkappa a$ will be much smaller still.

Mathematics. — *Zur projektiven Differentialgeometrie der Regelflächen im R_4.* (Siebente Mitteilung.) Von R. WEITZENBÖCK und W. J. BOS.

(Communicated at the meeting of November 29, 1941.)

Im Anschluss an die vorige Mitteilungen zählen wir hier die projektiven Ausartungen der $F_2{}^3$ im R_4 auf.

§ 22.

Die am Schluss der ersten Mitteilung gegebene Klassifikation ergibt:

$$M'_{02} \not\equiv 0,\ H \equiv 0,\ F_2^3 \text{ in einem } R_3(v').$$

Die Geraden p_{ik} und q_{ik} sind Erzeugenden der $F_2{}^3$; d.h.: $\varrho\, v'_i = (p^2 q^2)_i$.
Weiter haben wir (Vgl. (196)):

$$H \equiv (pu')\,(pb^2\, b_1^2) + \tfrac{2}{3}\, t\,(qu')\,(qa^2\, a_1^2) \equiv 0.$$

Oder:

$$(pu')\,(pb^2\, b_1^2) = 0$$

$$(qu')\,(qa^2\, a_1^2) = 0.$$

Die Gerade p liegt also im Brennraum des Ebenenkomplexes K_b und q liegt im Brennraum des Komplexes K_a.

Aus den Gleichungen $M'_{pa} = 0$ und $M'_{qb} = 0$ (194) erhalten wir: p resp. q ist ein singulärer Strahl des Komplexes K_a resp. K_b und liegt also auch im Brennraum des Komplexes K_a resp. K_b.

Der Raum v' ist also der Brennraum der beiden Komplexe K_a und K_b:

$$\varrho\, v'_i = (p^2\, q^2)_i = \lambda\,(a^2\, a_1^2)_i = \mu\,(b^2\, b_1^2)_i. \quad . \quad . \quad . \quad . \quad . \quad (212)$$

Wir wählen das Koordinatensystem derartig, dass der Raum v' die Gleichung $u'_5 = 0$ hat. Dann ist also $p_{i5} = 0$, $q_{i5} = 0$, $a_{i5} = 0$, $b_{i5} = 0$.

Dies gibt die folgende Darstellung der $F_2{}^3$ im Raume v':

$$0_{ik}(t) = p_{ik} + t\,.\,a_{ik} + t^2\,.\,b_{ik} + t^3\,.\,q_{ik} \quad (i, k = 1, 2, 3, 4)\,. \quad (213)$$

Aus $M'_{00} = 0\{t\}$ ergeben sich, analog (194), die Gleichungen:

$$\left.\begin{array}{ll} (p^2\, p_1^2) = 0 & (p^2\, a^2) = 0 \\ 2\,(p^2\, b^2) + & (a^2\, a_1^2) = 0 \\ (p^2\, q^2) + & (a^2\, b^2) = 0 \\ (b^2\, b_1^2) + 2\,(a^2\, q^2) = 0 & \\ (b^2\, q^2) = 0 & (q^2\, q_1^2) = 0 \end{array}\right\} \quad . \quad . \quad . \quad . \quad . \quad . \quad (214)$$

Wir betrachten die vier Strahlenkomplexe K_p, K_a, K_b und K_q.

Wenn

$$\triangle = |A_{ik}| \neq 0, \text{ (wo } A_{11} = (p^2 p_1^2),\ A_{12} = (p^2 a^2), \text{ u. s. w.)} \quad . \quad (215)$$

gibt es zwei Geraden L und M, welche allen vier Strahlenkomplexen angehören.

Die Komplexe K_p und K_q sind speziell; also schneiden p und q die Geraden L und M. L und M sind Leitlinien der F_2^3, denn jede Erzeugende $0_{ik}(t)$ schneidet L und M.

Nennen wir die Schnittpunkte von p mit L und M resp. 1 und 3 und die Schnittpunkte von q mit L und M resp. 2 und 4, dann ist:

$$p_{ik} = (13)_{ik},\ q_{ik} = (24)_{ik},\ L_{ik} = (12)_{ik},\ M_{ik} = (34)_{ik}.$$

Die Umformung des Ausdrucks $(1234)\ (a^2 \pi^2)$ ergibt:

$$(1234)\,(a^2\,\pi^2) = -\,(a^2\,24)\,(13\,\pi^2) + (a^2\,23)\,(14\,\pi^2) + (a^2\,14)\,(23\,\pi^2).$$

Damit ist a_{ik} dargestelt als eine lineare Kombination der Linienkoordinaten $(13)_{ik}$, $(14)_{ik}$ und $(23)_{ik}$.

Ebenso erhält man b_{ik} als eine lineare Kombination der Grössen: $(24)_{ik}$, $(23)_{ik}$ und $(14)_{ik}$.

Dies führt zu folgendem Ansatz: Wir setzen:

$$y = 1 + t\,.\,2 \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (L)$$

$$z = 3 + t\,(3 + 4) + t^2\,.\,4 \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (M)$$

Die Geraden L und M sind dann ein-zweideutig auf einander bezogen.

Die Verbindungslinien entsprechender Punkte liefern die folgende Darstellung der F_2^3.

$$0_{ik}(t) = (13)_{ik} + \{(13)_{ik} + (14)_{ik} + (23)_{ik}\}\,t + \\ + \{(14)_{ik} + (23)_{ik} + (24)_{ik}\}\,t^2 + (24)_{ik}\,t^3.$$

Eine lineare Parametertransformation gestattet uns auch die Darstellung:

$$y = 1 + t\,.\,2 \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (L)$$

$$z = 3 + t^2\,.\,4 \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (M)$$

und also

$$0_{ik}(t) = (13)_{ik} + t\,.\,(23)_{ik} + t^2\,(14)_{ik} + t^3\,(24)_{ik}\,. \quad . \quad . \quad (216)$$

Man kann also in diesem Falle mit Hilfe einer Parametertransformation die F_2^3 in eine Gestalt bringen bei welcher die Komplexe K_a und K_b speziell sind.

Wenden wir uns jetzt dem Falle zu, dass die vier Komplexe K_p, K_a, K_b, und K_q nur eine Gerade L gemeinsam haben.

Dann ist: $\triangle = |A_{ik}| = 0$ (vgl. 215)

Also nach (214):

$$\triangle \equiv \begin{vmatrix} 0 & 0 & -\frac{1}{2}(a^2\,a_1^2) & -(a^2\,b^2) \\ 0 & (a^2\,a_1^2) & (a^2\,b^2) & -\frac{1}{2}(b^2\,b_1^2) \\ -\frac{1}{2}(a^2\,a_1^2) & (a^2\,b^2) & (b^2\,b_1^2) & 0 \\ -(a^2\,b^2) & -\frac{1}{2}(b^2\,b_1^2) & 0 & 0 \end{vmatrix} = 0$$

$$\triangle = \{(a^2\,b^2)^2 - \tfrac{1}{4}(a^2\,a_1^2)\,(b^2\,b_1^2)\}^2 = 0.$$

Also:

$$4\,(a^2\,b)^2 - (a^2\,a_1^2)\,(b^2\,b_1^2) = 0. \quad . \quad . \quad . \quad . \quad . \quad . \quad (217)$$

Die weitere Rechnung wird einfacher wenn wir die homogenen Grössen a_{ik} und b_{ik} derartig wählen dass: $(a^2 a_1^2) = 2(a^2 b^2)$ und also mit (217):

$$(a^2\,a_1^2) = 2\,(a^2\,b^2) = (b^2\,b_1^2) = 2f$$

ist.

War z.B. ursprünglich $\frac{(a^2\,a_1^2)}{2\,(a^2 b^2)} = c$ dann nehmen wir:

$$\hat{p}_{ik} = \frac{p_{ik}}{c^2}\,;\ \hat{a}_{ik} = \frac{a_{ik}}{c}\,;\ \hat{b}_{ik} = b_{ik}\,;\ \hat{q}_{ik} = c\,.\,q_{ik}\,;\ \hat{t} = \frac{t}{c}$$

es ist dann

$$\hat{0}_{ik}\,(t) = \frac{0_{ik}\,(t)}{c^2} \quad \text{und} \quad \frac{(\hat{a}^2\,\hat{a}_1^2)}{2\,(\hat{a}^2\,\hat{b}^2)} = 1.$$

Die Gleichung der doppelten Leitlinie ist dann gegeben durch:

$$\begin{vmatrix} 0 & 0 & -f & -f & (p\pi')^2 \\ 0 & 2f & f & -f & (a\pi')^2 \\ -f & f & 2f & 0 & (b\pi')^2 \\ -f & -f & 0 & 0 & (q\pi')^2 \\ (p\pi')^2 & (a\pi')^2 & (b\pi')^2 & (q\pi')^2 & 0 \end{vmatrix} = 0.$$

Dies gibt:

$$2\,f^3\,[(p\pi')^2 + (a\pi')^2 + (b\pi')^2 + (q\pi')^2]^2 = 0.$$

Die Leitlinie L ist also:

$$L_{ik} = p_{ik} + a_{ik} + b_{ik} + q_{ik}$$

d.h. L_{ik} ist die Erzeugende $t = 1$.

Nennen wir die Schnittpunkte von L_{ik} mit p und q resp. 1 und 2, und wählen wir auf p einen von 1 verschiedenen Punkt 3.

Wir legen dann durch den Punkt 3 und die Erzeugende q $(t = \infty)$ eine Ebene $(w'x) = (3q^2x) = 0$ und schneiden diese Ebene mit den Erzeugenden $0_{ik}(t)$ (*Gl* (213)). Also:

$$(3\,q^2\,0)\,(0\,u') = (3\,q^2\,1)\,(3\,u') + (3\,q^2\,a)\,(au')\,t + \\ + (3\,q^2\,b)\,(b\,u')\,t^2 + (3\,q^2\,q_1)\,(q_1\,u')\,t^3 = 0.$$

Da der letzte Term verschwindet bekommen wir also einen Kegelschnitt, der den Punkt 2 enthält und dessen Punkte durch die Erzeugenden $0_{ik}(t)$ projektiv auf die der Leitlinie L bezogen sind.

Die Fläche $F_2{}^3$ kann man also in diesem Falle darstellen mit Hilfe einer nicht singulären (1, 1) Korrespondenz zwischen den Punkten einer Gerade L und den Punkten eines Kegelschnittes K, wobei L und K einen Punkt gemeinsam haben.

Wir setzen:

$$y = 1 + t \,.\, 2 \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (L)$$

$$z = 3 + t \,.\, (2{-}3{-}4) + t^2 \,.\, 4 \qquad (K \text{ enthält den Punkt } 2) \quad . \quad . \quad (K)$$

Also:

$$\left.\begin{aligned} 0_{ik}(t) = (13)_{ik} + t\{(12)_{ik} - (13)_{ik} - (14)_{ik} + (23)_{ik}\} + \\ + t^2\{(14)_{ik} - (23)_{ik} - (24)_{ik}\} + t^3 (24)_{ik} \end{aligned}\right\} \quad . \quad . \quad (219)$$

Der Parameterwert $t = 1$ gibt die Leitlinie $L_{ik} = (12)_{ik}$.

Wie zu erwarten war haben wir also die zwei bekannten Typen (216) und (219) der nicht ausgearteten und nicht abwickelbaren Regelflächen $F_2{}^3$ im R_3 gefunden.

§ 23.

Die Fläche $F_2{}^3$ (192) ist abwickelbar wenn $M'_{02} \equiv 0$, also wegen (195) wenn:

$$\left.\begin{aligned} M'_{pb} &= 0 \\ M'_{pq} &= 0 \\ M'_{aq} &= 0 \end{aligned}\right\} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (220)$$

Mit (194) haben wir in diesem Falle:

$$M'_{aa} = 0, \; M'_{ab} = 0, \; M'_{bb} = 0.$$

Die Grössen a_{ik} und b_{ik} sind Linienkoordinaten und jede der vier Geraden p, a, b, und q schneidet die drei anderen.

Wir haben also die Möglichkeiten:

1. Es gibt keinen Punkt, welcher den vier Geraden angehört.

Die vier Geraden liegen in einer Ebene und man erhält für die $0_{ik}(t)$ die Darstellung:

$$0_{ik}(t) = (12) + t \,.\, (23) + t^2 \,.\, (34) + t^3 \,.\, (41).$$

1, 2, 3, 4 sind allgemeine Punkte der Ebene.

Die Geraden $0_{ik}(t)$ bilden hier eine ebene Kurve dritter Klasse.

2. Die vier Geraden haben einen Punkt 1 gemeinsam und liegen nicht in einem R_3. Also:

$$0_{ik}(t) = (12) + t \,.\, (13) + t^2 \,.\, (14) + t^3 \,.\, (15) \quad (\text{mit } (12345) \neq 0).$$

Die Erzeugenden bilden einen Kegel dritter Ordnung im R_4.

3. Die vier Geraden haben einen Punkt 1 gemeinsam und liegen in einem R_3, aber nicht in einer Ebene:

$$0_{ik}(t) = (12) + t \,.\, (13) + t^2 \,.\, (14) + t^3 \,.\, (15) \quad (\text{mit } (12345) = 0).$$

Die 0_{ik} bilden einen Kegel dritter Ordnung im R_3.

4. Die vier Geraden haben einen Punkt 1 gemeinsam und liegen in einer Ebene. Die Erzeugenden bilden einen Geradenbüschel.

Zum Schluss noch einige Bemerkungen.

1. $(pu')\,(pb^2b_1{}^2) = 0$ genügt zum Verschwinden des Ausdruckes H (196). Denn wir

erhalten dann aus (194): $(pu')\,(pa^2q^2) = 0$ oder $(au')\,(ap^2q^2) = 0$. Zu Beginn des Paragraphen 22 zeigten wir:

Wenn $(pu')\,(pb^2b_1^2) = 0$ dann ist $(p^2q^2)_i = \mu(b^2b_1^2)_i$ (Gl. (212)). Also auch $(au')\,(ab^2b_1^2) = 0$. Also: $(au')\,(aa^2q^2) = 0$, oder: $(qu')\,(qa^2a_1^2) = 0$.

Dies gibt: $H \equiv 0$.

2. Weiter kann man leicht zeigen:

$M'_{aa} = 0$ genügt zum Verschwinden des Ausdruckes H; und auch wenn $M'_{pq} = 0$, ist $H \equiv 0$.

Betrachten wir die Formeln (217) und (220) dann haben wir also:

$$M'_{aa} = 0,\ M'_{pq} \neq 0, \qquad F_2{}^3 \text{ im } R_3. \text{ Typus I (216)}$$

$$M'_{aa} = 0,\ M'_{pq} = 0,\ M'_{bb} \neq 0 \qquad F_2{}^3 \text{ im } R_3. \text{ Typus II (219)}$$

$$M'_{aa} = 0,\ M'_{pq} = 0,\ M'_{bb} = 0 \qquad F_2{}^3 \text{ abwickelbar.}$$

Medicine. — *Some remarks on symmetrical hearing remnants.* By A. DE KLEYN.

(Communicated at the meeting of Juny 28, 1941.)

In 1936 LANGENBECK [1]) communicated the result of his audiometric examinations and pointed to the fact that in most cases of hereditary deaf-mutism he found symmetrical hearing remnants. He also spoke of symmetrical hearing remnants when the patients had no hearing residues at all on either side. From the results of his investigations he further concluded that in doubtful cases the presence of symmetrical hearing rests furnished strong evidence of an hereditary genesis, especially when it is combined with a vestibular apparatus which can be easily stimulated. LANGENBECK was so convined of the truth of his conclusion that he thought that his denomination "Symmetriegesetz der erblichen Taubheit" was not putting the matter too strongly.

The difficulty that in some cases deafness with symmetrical hearing rests only develops in connection with a disease, e.g. infectious diseases, was solved by him by accepting the presence of a hereditary inferiority of the hearing organs of these patients and supposing that these were damaged identically at both sides by the toxines of the infectious germs. On the other hand he did not think that this could be the case in persons who had no hereditary transmission, for he wrote: "Es wird jedenfalls durch die obengenannten Beobachtungen noch in keiner Weise bewiesen, dasz eine toxische auf dem Blutwege erfolgende Schädigung beim Erbgesunden auf beiden Ohren symmetrisch gleiche Hörausfälle erzeugen kann".

As an example of symmetrical hearing remnants in two types of hereditary deafness the following observations made by myself are recorded.

Fig. 1 concerns a boy, 20 years of age, with congenital hereditary deaf-mutism. The

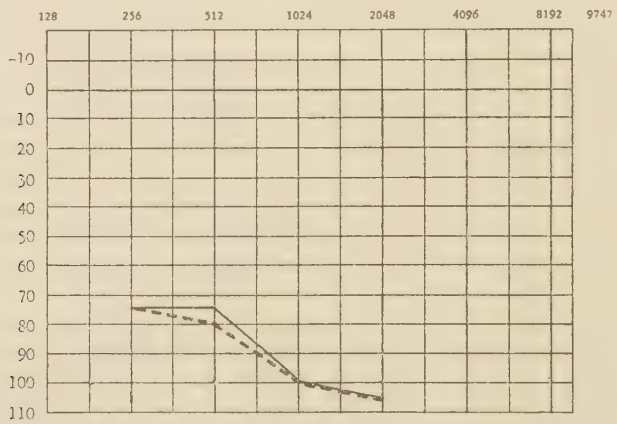

Fig. 1.

heredity was clear from the fact that in his family more cases of deaf-mutism were found. Examination with the audiometer (Western Electric) revealed that only the tones with 256, 512, 1024 and 2048 [2]) frequencies were heard while the minimum receptible for these tones was about the same bilaterally [3]), which means that this patient had symmetrical hearing remnants.

[1]) LANGENBECK, B., Ztschr. f. Hals–Nasen–Ohrenheilk., **39**, 223, 286 (1936).

[2]) In all audiograms ———— means the perceptibility of air-conduction of the right ear, — — — — of the left ear.

[3]) Differences of 5 decibels are by this audiometer within the fault of perceptibility.

Fig. 2 concerns another type of hereditary deafness, the so-called hereditary degenerative internal-ear-deafness, developing a shorter or longer time after birth and causing a variable degree of deafness. In this case the hearing remnants of a girl, 15 years of age, were completely symmetrical. Patient was an only child and suffered from an increasing deafness since she was 6 years of age. The inheritability of deafness was out of the question: both her father and her uncle (40 and 41 years of age) showed the same type of deafness, both had almost symmetrical hearing rests.

Not in all cases of true hereditary deafness, symmetrical hearing remnants are found, however. LANGENBECK himself observed such cases. According to him these are patients whose audiograms only showed a symmetry for the higher, not for the lower frequencies. In many of these cases the asymmetry is supposed to be caused by a former otitis of one of the ears. Yet LANGENBECK speaks of "Regelfälle" as far as his "Symmetriegesetz" is concerned, with slight disturbances in the audiogram of one ear: "Die Abweichungen müssen aber durch überstandene Erkrankungen und Rückstände zu erklären sein, insbesondere müssen linke und rechte Hörkurve auf grosse Strecken identisch verlaufen und vor allem an den oberen Hörgrenze wieder zusammenlaufen".

We did not meet with distinct cases of this type among our own patients.

On the other hand we found that symmetrical hearing remnants in true cases of hereditary deafness are often absent and this to a much greater extent than is recorded of the patients of LANGENBECK.

Fig. 3*a* and 3*b* represent a typical example. Four children of deaf-mute parents are also deaf-mute and a brother of their father as well. Three of these 4 children were examined. A boy, 12 years of age, had no hearing residue at all and could be considered to belong to the symmetry-cases of LANGENBECK on the understanding that one accepts that bilateral absence is also a symmetry. One brother of 8 years, however, perceived only tones of 1024 frequencies with his right ear, whereas the left ear heard tones of 256, 512 and 1024 frequencies (fig. 3*a*). Another brother 6 years of age, had completely asymmetric audiograms: with the right ear tones of 128, 256 and 512 frequencies, with the left one those of 256, 512, 1024 and 2048 frequencies were heard, so that the symmetry-law of LANGENBECK is altogether out of the question.

The audiograms shown in fig. 4 and 5 are also completely asymmetric. Fig. 4 is of a girl, 13 years of age, with true hereditary congenital deaf-mutism and shows a pronounced asymmetry especially in the region of the middle and higher tones. Fig. 5 shows the audiograms of a deaf-mute girl of 14 years. Two brothers and one sister were also deaf-mute, six other children had good hearing. The sister of her mother also suffered from deaf-mutism. The complete asymmetry of these audiograms needs no further description.

To these examples one could add many others: in themselves, however, they form no considerable objection to the examination of inheritability. They only prove that, if symmetrical hearing remnants were absent, one might not conclude that heredity plays not a part.

The main question on the contrary, is, if one is justified in accepting a hereditary genesis in those cases where symmetrical hearing residues are found. LANGENBECK holds this opinion although it must be granted that his conclusions are extremely cautious.

However, many cases observed by us in the last years, are contrary to this conception.

Fig. 6 shows the completely symmetrical hearing remnants of a 14 years old deaf-mute boy. When he was $1\frac{1}{2}$ year of age, speaking strikingly well for his age, he suffered from a parotitis after which he became almost completely deaf. In this case the symmetry of the hearing rests, as shown in the figure, is very obvious. Both parents of this child had a good hearing, while, as far as could be gathered, no deafness was found in the family. In my opinion there is no reason therefore to accept the possibility of a hereditary genesis.

No more is this the case in a boy of 14 years of age, whose audiograms are shown in fig. 7. He belonged to a family of 5 children of which 3 died when very young. This patient showed symptoms of a congenital lues; the Wassermann reaction was strongly positive. A brother of 15 years had no luetic symptoms, although in his case the

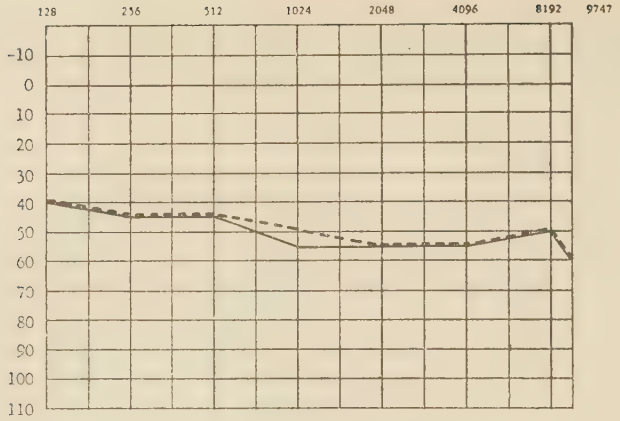

Fig. 2.

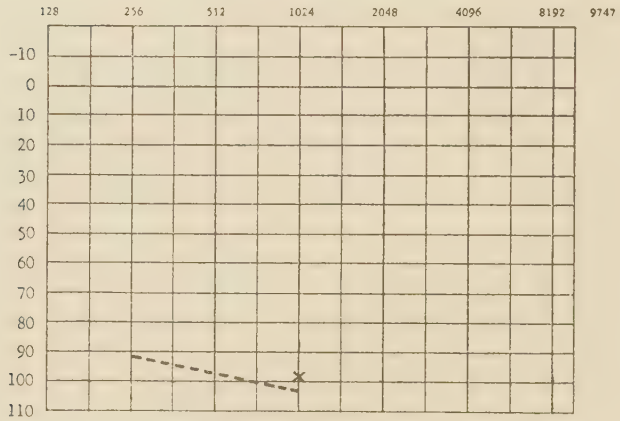

Fig. 3a.

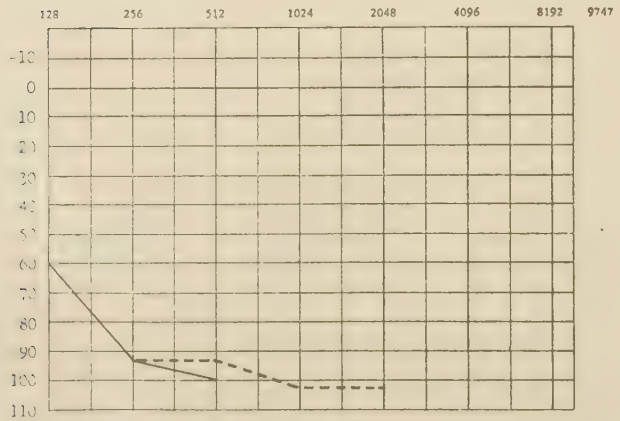

Fig. 3b.

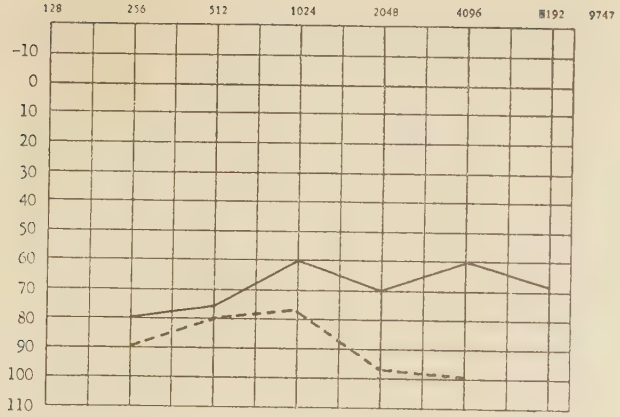

Fig. 4.

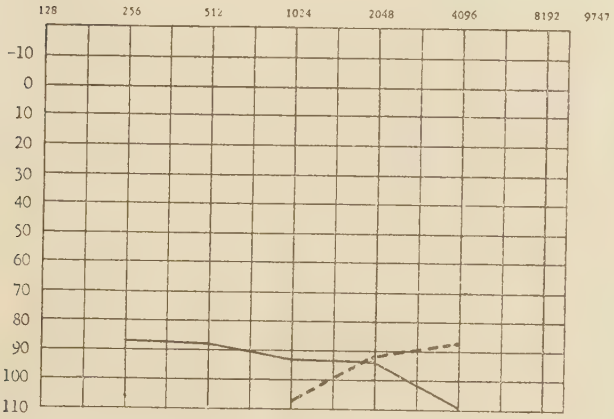

Fig. 5.

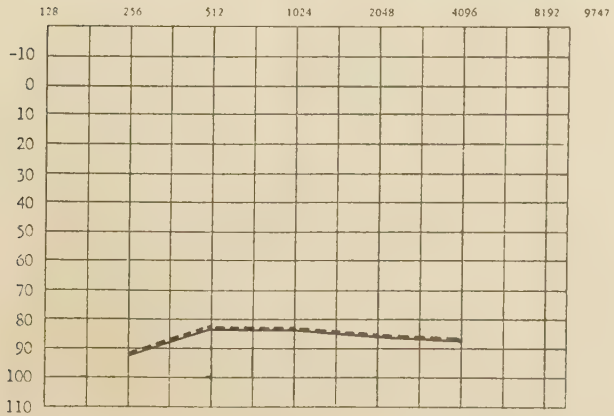

Fig. 6.

Wassermann reaction was also strongly positive. In the family no deaf-mutism or a severe degree of deafness occurred. So in this case the cause of the deaf-mutism must most probably be ascribed to lues. However, the hearing remnants, shown in fig. 7, were completely symmetric.

Probably to save his "Symmetriegesetz" for such cases, LANGENBECK gave the following explanation which is not convincing: "Findet man aber derartige Anamnesen in unbelasteten Familien, und bei Prüflingen, die sich auch als Regelfälle im Sinne des Symmetriegesetzes erweisen, so muss man auch daran denken, dass in der Familie solcher Prüflinge die Manifestationstendenz der erblichen Taubheit so gering sein kann, dass es erst des Anstosses einer toxischen Schädigung bedarf, um die Erbkrankheit zum Durchbruch kommen zu lassen."

To me it does not seem difficult to find with such reasoning an explanation of any diversity of his "Law of symmetry".

The case described below, however, would have brought LANGENBECK himself into difficulties.

A young woman had suffered from a repeated otitis from her 3rd year. At the right side a radical operation had to be performed, at the left a conservative treatment was sufficient. In the course of years both ears became dry, but the acuity of hearing had suffered much by all the inflammations so that when patient was 33 years of age, she suffered from a pronounced deafness. The audiograms of both ears, however, were completely symmetric (fig. 8). In the family no cases of deafness occurred.

LANGENBECK discussed the question whether it is possible that symmetrical hearing rests can be left after an inflammation and thought this must be denied: "Denn dass eine auf entzündlichem Wege entstandene Schwerhörigkeit zufällig gerade auf beide Ohren genau den gleichen Hörausfall hervorrufen sollte, wird man nicht annehmen können." Yet in our patient mentioned above such symmetrical hearing hemnants were found.

> *Summarizing the above mentioned facts, one has to assume that although in many cases of hereditary deafness symmetrical hearing remnants are found, it is neither permitted to deny such a hereditary genesis when such a symmetry is absent, nor to accept a hereditary genesis when a symmetry is present.*

The opinion of SCHWARZ [1]) that a deafness "selbst nach exakter Untersuchung und nach eingehender Beobachtung aus dem Befund allein nicht mit der erforderlichen Sicherkeit als erbbedingt oder umweltbedingt auseinander zuhalten ist" remains right and we cannot support the idea of LANGENBECK: "Somit kann man die These voll und ganz aufrechterhalten, dass die Symmetrie der Hörreste bei erblicher Taubheit, bezw. Schwerhörigkeit, ein brauchbares Symptom für die Beurteilung der Erblichkeit ist."

From the above mentioned it is evident that symmetrical hearing remnants are only of a relative value for the investigation of heredity. In spite of this fact it has been a great merit of LANGENBECK that he has emphasized the above mentioned symmetry.

In his communication he pointed also to the fact, already known for a long time, that audiograms of individuals with a normal hearing are generally of a striking symmetry and that, if these, having a certain pitch, are somewhat divergent from the standard audiogram of normal individuals, this divergence is in many cases of precisely the same extent for both ears. Sometimes the same holds good for the audiograms of several members of a family so that the term "Familienhörkurve" would not be exaggerated.

In patients suffering from presbyacusis too a striking symmetry in the audiograms of both ears is often found. A typical example is seen in fig. 9. It concerns a man of 74 years of age, who had never had any ear disease, so that the disturbances in the audiograms had to be ascribed to his age.

LANGENBECK tried to explain the symmetry by accepting the same demolition both for

[1]) LANGENBECK, cit. blz. 223.

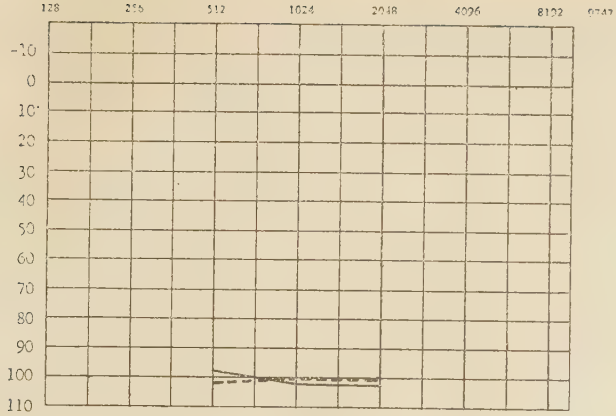

Fig. 7.

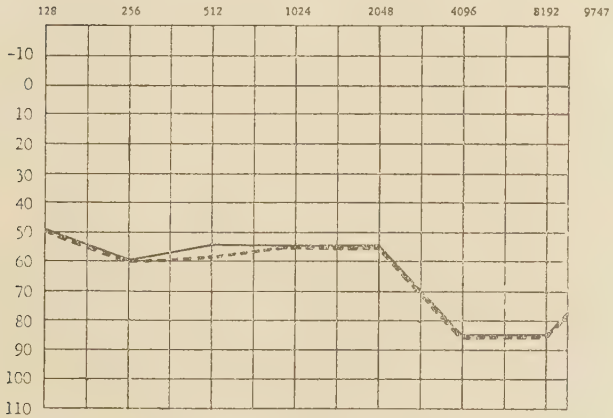

Fig. 8.

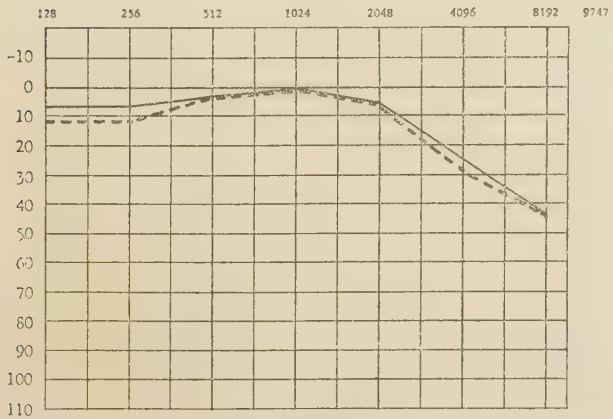

Fig. 9.

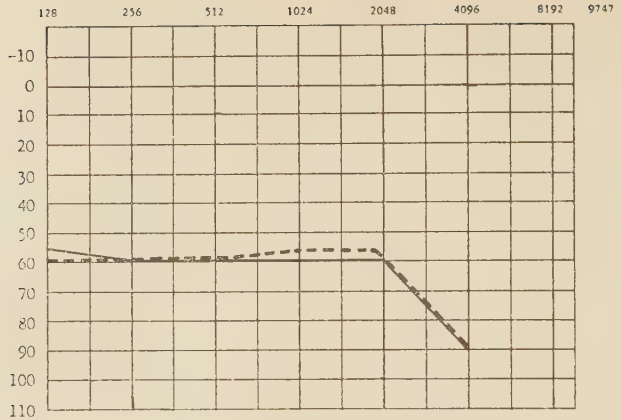

Fig. 10.

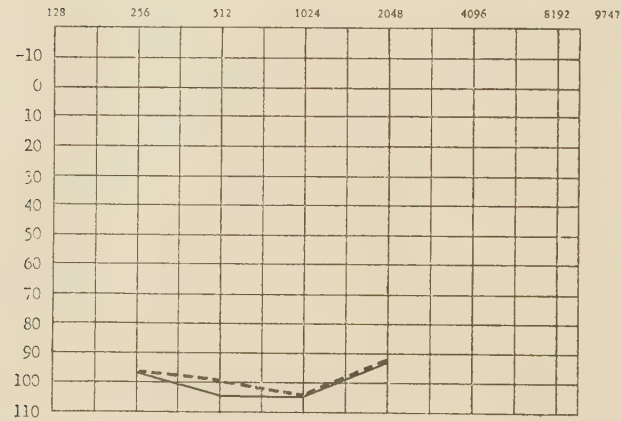

Fig. 11.

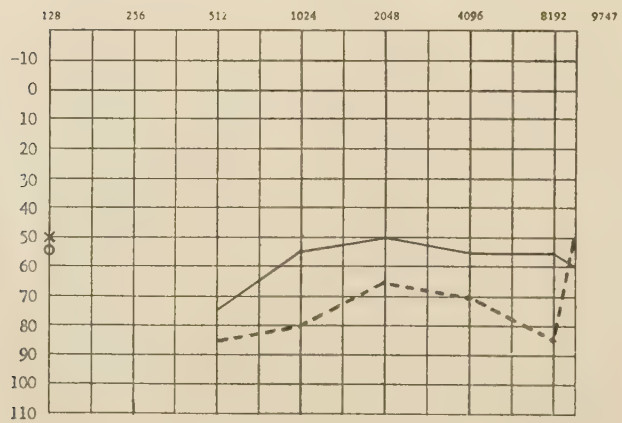

Fig. 12.

the left and the right ear, which as he says is completely determined by hereditary endogenous factors. In my opinion it is not necessary to think especially of hereditary factors; it may as well be assumed that the symmetry develops owing to constitutional non-hereditary factors, which, although varying individually, are present to the same extent at the left and at the right. So when both ears are damaged by some agens, either by an infectious disease (fig. 6 and 7) or by an inflammation, it is not strange that both ears loose an equal part of their function and that another more resistent part is kept. The result is that symmetrical hearing remnants are found, which as far as their extension is concerned, can differ very much individually.

Some observations from the Otorhinological Clinic of Amsterdam revealed the remarkable fact that these symmetrical hearing remnants can also develop when the function of the ears is not lost simultaneously but with intervals of years.

Some observations from the Otorhinological Clinic of Amsterdam revealed the remarkable fact that these symmetrical hearing remnants can also develop when the function of the ears is not lost simultaneously but with intervals of years.

I. The medical history of the above mentioned patient (fig. 8) points already in this direction. This woman, suffering from a bilateral otitis media, had been deaf in the right ear for many years and it had undergone a radical operation. The hearing of the left ear was sufficient. However, this acuity also gradually diminished and reached a level completely identical to that of the other side.

II. Another example is found in a woman, 65 years of age, who, after a pregnancy, became deaf in one ear. Many years after this, after an attack of malaria, which was treated with large doses of chinine, the other ear also became suddenly deaf: after this the hearing rests were completely symmetrical at both sides (fig. 10). Further acoustic and otoscopic examination revealed a bilateral middle ear-deafness.

III. Fig. 11 shows the symmetrical hearing residue of a man of 28 years of age, who, as long as he remembers, had been deaf in his left ear. In his family deafness was unknown. He was a hair-dresser but had never met with any difficulty in commerce because, as he said himself, his right ear was completely sound. His family-doctor had also never noticed any deafness. Some days before his admission to the clinic he awoke completely deaf. Extensive internal, neurological and serological examination revealed no cause for the deafness. It was very remarkable that also in this case the hearing rests were completely symmetrical.

IV. The last example is that of a girl of 9 years, the youngest of 5 children of whom two are very deaf and have symmetrical hearing rests, whereas the two others have a good hearing. We have known both the patient and the whole family for years.

The left ear of the patient was very deaf, the right one on the contrary completely normal. Some days before admission to the clinic patient stumbled and fell. Immediately after this she was very deaf in the right ear, although she had not lost consciousness after her fall. Fig. 12 shows the audiograms of both ears. Here a true symmetry was altogether out of the question, but it was a remarkable fact that where a tone of 128 frequencies was heard at both sides (x and o on the figure) it was impossible to perceive a tone of 256 frequencies neither with the right nor with the left ear, although the greatest intensity that the audiometer could supply was used. On the other hand tones above this frequency were heard with both ears. So for both ears only the perception of a tone of 256 frequencies was failing.

In my opinion all these observations are best explained by accepting the fact that a certain and *bilateral equal* part of the function of the hearing organs is more vulnerable than the remaining less vulnerable part and that if both ears are severely damaged by some cause, it may happen that only this more vulnerable part is put out of function. The remaining part will then manifest itself in symmetrical hearing remnants of the right and the left ear.

Mathematics. — *Stark konvergente Entwicklungen für die vollständigen elliptischen Integrale erster und zweiter Art.* III. Von S. C. VAN VEEN. (Communicated by Prof. J. G. VAN DER CORPUT.)

(Communicated at the meeting of November 29, 1941.)

§ 3. *Die Entwicklung des vollständigen elliptischen Integrales zweiter Art.* Das vollständige elliptische Integral zweiter Art wird definiert durch

$$E(k) = \int_0^{\frac{\pi}{2}} \sqrt{1-k^2 \sin^2\varphi}\, d\varphi = \frac{\pi}{2} F(-\tfrac{1}{2}, +\tfrac{1}{2}; 1; k^2). \quad . \quad (37)$$

Wenn k_n, a_n und b_n die Hilfsgrössen aus § 1 bezeichnen, so gilt

Hilfssatz 2: *Für ganzes* $n \geqq 1$ *ist*

$$2E(k_n) = \frac{a_{n-1}}{a_n} E(k_{n-1}) + \frac{b_{n-1}}{a_n} K(k_{n-1}) \quad . \quad . \quad . \quad . \quad (38)$$

$$= \frac{a_{n-1}}{a_n} E(k_{n-1}) + \frac{b_n^2}{a_n^2} K(k_n). \quad . \quad . \quad . \quad . \quad . \quad (38')$$

*(*LANDEN*schen Transformation für vollständige elliptische Integrale zweiter Art).*

Beweis: Wendet man auf

$$E(p, \beta) = \int_0^{\beta} \sqrt{1-p^2 \sin^2\varphi}\, d\varphi$$

die LANDENsche Transformation

$$tg\,\varphi = \frac{\sin 2\omega}{p + \cos 2\omega}$$

an, so findet man

$$\cos\varphi = \frac{p + \cos 2\omega}{\sqrt{1 + 2p\cos 2\omega + p^2}};$$

$$\sqrt{1-p^2 \sin^2\omega} = \frac{1 + p\cos 2\omega}{\sqrt{1 + 2p\cos 2\omega + p^2}};$$

$$\frac{d\varphi}{d\omega} = \frac{2(1 + p\cos 2\omega)}{1 + 2p\cos 2\omega + p^2}.$$

Hieraus ergibt sich

$$\left.\begin{aligned} E(p,\beta)+p\,.\,\sin\beta &= \int_0^{\beta}(\sqrt{1-p^2\sin^2\varphi}+p\cos\varphi)\,d\varphi = \\ &= 2\int_0^{\beta_1}\frac{1+p\cos 2\omega}{\sqrt{1+2p\cos 2\omega+p^2}}\,d\omega = \frac{2}{1+p}\int_0^{\beta_1}\frac{1+p-2p\sin^2\omega}{\sqrt{1-\frac{4p}{(1+p)^2}\sin^2\omega}}\,d\omega = \\ &= (1+p)\int_0^{\beta_1}\sqrt{1-\frac{4p}{(1+p)^2}\sin^2\omega}\,d\omega+(1-p)\int_0^{\beta_1}\frac{d\omega}{\sqrt{1-\frac{4p}{(1+p)^2}\sin^2\omega}} = \\ &= (1+p)\,.\,E\left(\frac{2\sqrt{p}}{1+p},\beta_1\right)+(1-p)\,F\left(\frac{2\sqrt{p}}{1+p},\beta_1\right), \end{aligned}\right\} \quad (39)$$

mit

$$tg\,\beta = \frac{\sin 2\beta_1}{p+\cos 2\beta_1}$$

Wenn ω das Intervall $\left(0,\frac{\pi}{2}\right)$ durchläuft, so durchläuft φ das Intervall $(0,\pi)$ und man findet aus (39)

$$\left.\begin{aligned} E(p,\pi) &= (1+p)\,.\,E\left(\frac{2\sqrt{p}}{1+p},\frac{\pi}{2}\right)+ \\ &+(1-p)\,F\left(\frac{2\sqrt{p}}{1+p},\frac{\pi}{2}\right) = (1+p)\,E\left(\frac{2\sqrt{p}}{1+p}\right)+(1-p)\,K\left(\frac{2\sqrt{p}}{1+p}\right). \end{aligned}\right\} \quad (40)$$

Setzt man

$$\frac{2\sqrt{p}}{1+p} = k_{n-1}$$

also, wegen (9)

$$p = \frac{1-\sqrt{1-k_{n-1}^2}}{1+\sqrt{1-k_{n-1}^2}} = k_n,$$

und, wegen (6) und (7)

$$1+p = 1+k_n = \frac{a_{n-1}}{a_n},\quad 1-p = 1-k_n = \frac{b_{n-1}}{a_n},$$

so geht (40) über in (38), und wegen (12) und (1) in (38').

Satz VI: *Für ganzes $n\geqq 1$ ist*

$$E(k_1) = 2^{n-1}\,a_n\,E(k_n)-\frac{1}{2\,a_n}\,K(k_n)\,.\sum_{m=2}^{n}\,.\,2^{m-1}\,b_m^2, \quad . \quad . \quad (41)$$

und für ganzes $n \geqq 2$ ist

$$E(k_1) = \pi a_n \cdot 2^{n-3} \cdot F(-\tfrac{1}{2}, -\tfrac{1}{2}; 1; k_n^2) - \frac{\pi}{2 a_n} F(\tfrac{1}{2}, \tfrac{1}{2}; 1; k_n^2) \cdot \sum_{m=2}^{n-1} 2^{m-2} \cdot b_m^2. \quad (42)$$

Beweis: Wegen (38) ist

$$\sum_{m=2}^{n} \{2^m a_m E(k_m) - 2^{m-1} a_{m-1} E(k_{m-1})\} = \sum_{m=2}^{n} 2^{m-1} b_{m-1} K(k_{m-1}),$$

also, wegen (12) und (1)

$$2^n a_n E(k_n) - 2 E(k_1) = K(k_1) \cdot \sum_{m=2}^{n} 2^{m-1} b_{m-1} \cdot a_{m-1} = K(k_1) \sum_{m=2}^{n} 2^{m-1} b_m^2 = \frac{K(k_n)}{a_n} \sum_{m=2}^{n} 2^{m-1} b_m^2,$$

womit (41) bewiesen ist.

Für $n \geqq 2$ folgt aus (41)

$$E(k_1) = 2^{n-1} a_n E(k_n) - \frac{2^{n-2} b_n^2}{a_n} \cdot K(k_n) - \frac{K(k_n)}{a_n} \cdot \sum_{m=2}^{n-1} 2^{m-2} b_m^2 =$$

$$= 2^{n-2} \cdot a_n \{2 E(k_n) - (1 - k_n^2) K(k_n)\} - \frac{K(k_n)}{a_n} \sum_{m=2}^{n-1} 2^{m-2} b_m^2 \quad \text{(wegen (8))}$$

$$= 2^{n-3} a_n \left\{ \sum_{m=0}^{\infty} \frac{\Gamma(m-\frac{1}{2}) \cdot \Gamma(m+\frac{1}{2})}{(m!)^2} k_n^{2m} + (1-k_n^2) \sum_{m=0}^{\infty} \left(\frac{\Gamma(m+\frac{1}{2})}{m!} \right)^2 k_n^{2m} \right\}$$

$$- \frac{K(k_n)}{a_n} \sum_{m=2}^{n-1} 2^{m-2} b_m^2 = 2^{n-5} a_n \sum_{m=0}^{\infty} \left(\frac{\Gamma(m-\frac{1}{2})}{m!} \right)^2 k_n^{2m} - \frac{K(k_n)}{a_n} \cdot \sum_{m=2}^{n-1} 2^{m-2} b_m^2 =$$

$$= 2^{n-3} \pi a_n \cdot F(-\tfrac{1}{2}, -\tfrac{1}{2}; 1; k_n^2) - \frac{\pi}{2 a_n} F(\tfrac{1}{2}, \tfrac{1}{2}; 1; k_n^2) \cdot \sum_{m=2}^{n-1} 2^{m-2} b_m^2,$$

womit (42) bewiesen ist.

A. *k liegt nicht in der Nähe von 1.*

Aus (3) ergibt sich

$$\left.\begin{aligned} b_2^2 &= \cos\alpha, \\ b_2^2 + 2 b_3^2 &= \cos\alpha + 2\cos^2\frac{\alpha}{2} \sqrt{\cos\alpha} = \sqrt{\cos\alpha}\,(1 + \sqrt{\cos\alpha} + \cos\alpha), \\ b_2^2 + 2 b_3^2 + 4 b_4^2 &= \cos\alpha + 2\cos\frac{\alpha}{2} \sqrt[4]{\cos\alpha} \left(\sqrt{\cos\alpha} + \cos\frac{\alpha}{2} \sqrt[4]{\cos\alpha} + \cos^2\frac{\alpha}{2} \right) \end{aligned}\right\} \quad (43)$$

u. s. w.

Endlich ergibt sich aus (41), bzw. (42), und (43), für $n = 1, \ldots, 5$

$$E(k) = \frac{\pi}{2} F(-\tfrac{1}{2}, \tfrac{1}{2}; 1; \sin^2\alpha) \quad . \quad . \quad . \quad . \quad . \quad (E\,I)$$

$$E(k) = \frac{\pi \cos^2\frac{\alpha}{2}}{2} F\left(-\tfrac{1}{2}, -\tfrac{1}{2}; 1; tg^4\frac{\alpha}{2}\right); \quad . \quad . \quad . \quad (E\,II)$$

$$E(k)=\pi\left(\frac{1+\sqrt{\cos\alpha}}{2}\right)^2 F\left(-\tfrac{1}{2},-\tfrac{1}{2};1;\left(\frac{1-\sqrt{\cos\alpha}}{1+\sqrt{\cos\alpha}}\right)^4\right)-$$

$$-\frac{2\pi\cos\alpha}{(1+\sqrt{\cos\alpha})^2}\cdot F\left(\tfrac{1}{2},\tfrac{1}{2};1;\left(\frac{1-\sqrt{\cos\alpha}}{1+\sqrt{\cos\alpha}}\right)^4\right);\quad (E\,III)$$

$$E(k)=2\pi\left(\frac{\cos\frac{\alpha}{2}+\sqrt[4]{\cos\alpha}}{2}\right)^2 F\left(-\tfrac{1}{2},-\tfrac{1}{2};1;\left(\frac{\cos\frac{\alpha}{2}-\sqrt[4]{\cos\alpha}}{\cos\frac{\alpha}{2}+\sqrt[4]{\cos\alpha}}\right)^4\right)$$

$$-\frac{2\pi\sqrt{\cos\alpha}(1+\sqrt{\cos\alpha}+\cos\alpha)}{\left(\cos\frac{\alpha}{2}+\sqrt[4]{\cos\alpha}\right)^2} F\left(\tfrac{1}{2},\tfrac{1}{2};1;\left(\frac{\cos\frac{\alpha}{2}-\sqrt[4]{\cos\alpha}}{\cos\frac{\alpha}{2}+\sqrt[4]{\cos\alpha}}\right)^4\right);\quad (E\,IV)$$

$$E(k)=\frac{\pi}{4}\left(1+\sqrt{\cos\alpha}+2\sqrt{\cos\frac{\alpha}{2}\cdot\sqrt[8]{\cos\alpha}}\right)^2\cdot$$

$$\cdot F\left(-\tfrac{1}{2},-\tfrac{1}{2};1;\left(\frac{1+\sqrt{\cos\alpha}-2\sqrt{\cos\frac{\alpha}{2}\cdot\sqrt[4]{\cos\alpha}}}{1+\sqrt{\cos\alpha}+2\sqrt{\cos\frac{\alpha}{2}\cdot\sqrt[8]{\cos\alpha}}}\right)^4\right)$$

$$-8\pi\frac{\left\{\cos\alpha+2\cos\frac{\alpha}{2}\cdot\sqrt[4]{\cos\alpha}\cdot\left(\sqrt{\cos\alpha}+\cos\frac{\alpha}{2}\cdot\sqrt[4]{\cos\alpha}+\cos^2\frac{\alpha}{2}\right)\right\}}{\left(1+\sqrt{\cos\alpha}+2\sqrt{\cos\frac{\alpha}{2}\cdot\sqrt[8]{\cos\alpha}}\right)^2}\cdot$$

$$\cdot F\left(\tfrac{1}{2},\tfrac{1}{2};1;\left(\frac{1+\sqrt{\cos\alpha}-2\sqrt{\cos\frac{\alpha}{2}\cdot\sqrt[8]{\cos\alpha}}}{1+\sqrt{\cos\alpha}+2\sqrt{\cos\frac{\alpha}{2}\cdot\sqrt[8]{\cos\alpha}}}\right)^4\right);\quad (E\,V)$$

u. s. w.

Wegen

$$k_n\approx\frac{\sin^{2^{n-1}}\alpha}{2^{2^{n-2}}}$$

für kleine Werte von k, ist die Stärke der Konvergenz mit der der Reihen (K) zu vergleichen. Die Hauptglieder der letzten drei Reihen sind

$$E(k)\approx\frac{\pi}{4}\left\{\frac{(1+\sqrt{\cos\alpha})^4-8\cos\alpha}{(1+\sqrt{\cos\alpha})^2}\right\};\quad .\ .\ .\ .\ .\ .\quad (E\,III')$$

$$E(k)\approx\frac{\pi}{2}\left\{\frac{\left(\cos\frac{\alpha}{2}+\sqrt[4]{\cos\alpha}\right)^4-4\sqrt{\cos\alpha}(1+\sqrt{\cos\alpha}+\cos\alpha)}{\left(\cos\frac{\alpha}{2}+\sqrt[4]{\cos\alpha}\right)^2}\right\};\quad (E\,IV')$$

$$E(k) \approx \frac{\pi}{4}\left\{\frac{\left(1+\sqrt{\cos\alpha}+2\sqrt{\cos\frac{\alpha}{2}}\cdot\sqrt[8]{\cos\alpha}\right)^4-32\left(\cos\alpha+\right.}{\left.+2\cos\frac{\alpha}{2}\sqrt[4]{\cos\alpha}\cdot\left(\sqrt{\cos\alpha}+\cos\frac{\alpha}{2}\sqrt[4]{\cos\alpha}+\cos^2\frac{\alpha}{2}\right)\right)}{\left(1+\sqrt{\cos\alpha}+2\sqrt{\cos\frac{\alpha}{2}}\cdot\sqrt[8]{\cos\alpha}\right)^2}\right\}. \qquad (E\,V')$$

FEHLERTABELLE.

α	$k_1 = \sin\alpha$	Fehler in		
		$E\,III'$	$E\,IV'$	$E\,V'$
$\leqq 45°$	$\leqq 0{,}70711$	$\leqq 1{,}9 \cdot 10^{-5}$	$\leqq 6{,}7 \cdot 10^{-11}$	$\leqq 8 \cdot 10^{-21}$
60°	0,86603	$2{,}7 \cdot 10^{-4}$	$1{,}5 \cdot 10^{-8}$	$4{,}2 \cdot 10^{-17}$
70°	0,93969	$1{,}4 \cdot 10^{-3}$	$3{,}9 \cdot 10^{-7}$	$3{,}4 \cdot 10^{-14}$
80°	0,98481	$7{,}5 \cdot 10^{-3}$	$1{,}4 \cdot 10^{-5}$	$4{,}6 \cdot 10^{-11}$
85°	0,99619	$2{,}3 \cdot 10^{-2}$	$1{,}4 \cdot 10^{-4}$	$4{,}4 \cdot 10^{-9}$
89°	0,99985	$9{,}5 \cdot 10^{-2}$	$2{,}9 \cdot 10^{-3}$	$2 \cdot 10^{-6}$
90°	1	$0{,}215 = 1 - \frac{\pi}{4}$	$0{,}215 = 1 - \frac{\pi}{4}$	$0{,}215 = 1 - \frac{\pi}{4}$

Die Reihen (E) konvergieren im ganzen Gebiet $0 \leqq \alpha \leqq \frac{\pi}{2}$, obgleich die Konvergenzstärke für $k = 1$ $\left(\alpha = \frac{\pi}{2}\right)$ ziemlich gering ist. Sie sind jedoch noch sehr gut verwendbar, wenn der Unterschied zwischen k und 1 gering ist.

Die Anwendung von (15) auf die Reihen E liefert fast gleich genaue Abschätzungen, mit ziemlich einfachen Hauptgliedern, zumal für $(E\,III)$ und $(E\,IV)$, nämlich

$$E(k) \approx \frac{\pi\sqrt[4]{\cos\alpha}}{2\cos\frac{\alpha}{2}}\left(1-\sqrt{\cos\alpha}+\cos\alpha\right); \qquad (E\,E\,III')$$

$$E(k) \approx \frac{\pi\sqrt{\cos\alpha}\left\{\sqrt{\cos\frac{\alpha}{2}}\cdot\sqrt[4]{\cos\alpha}\cdot(1+\sqrt{\cos\alpha})^2-(1+\sqrt{\cos\alpha}+\cos\alpha)\right\}}{\sqrt{\cos\frac{\alpha}{2}}\cdot\sqrt[8]{\cos\alpha}\cdot(1+\sqrt{\cos\alpha})}; \qquad (E\,E\,IV')$$

B. *k liegt in der Nähe von 1.*

Hilfssatz 3: *Wenn l_n und c_n die in § 2 definierten Hilfsgrössen bezeichnen, so ist für $n \geqq 1$*

$$E(k_1) = E\left(\sqrt{1-l_1^2}\right) = c_n\,E\sqrt{(1-l_n^2)} + K(k_1)\sum_{m=2}^{n} 2^{m-1}\,l_m\,c_m^2. \qquad (44)$$

Beweis: Wir setzen in (40)

$$p = \sqrt{1-l_{m-1}^2}$$

also

$$\frac{2\sqrt{p}}{1+p} = \sqrt{1-\left(\frac{1-\sqrt{1-l_{m-1}^2}}{1+\sqrt{1-l_{m-1}^2}}\right)^2} = \sqrt{1-l_m^2}.$$

(40) geht über in

$$2\,.\,E(\sqrt{1-l_{m-1}^2}) = (1+\sqrt{1-l_{m-1}^2})\,E(\sqrt{1-l_m^2}) + (1-\sqrt{1-l_{m-1}^2})\,K(\sqrt{1-l_m^2})$$

oder, wegen (33)

$$c_{m-1}\,E(\sqrt{1-l_{m-1}^2}) = c_m\,E(\sqrt{1-l_m^2}) + c_m\,l_m\,K(\sqrt{1-l_m^2}) =$$
$$= c_m\,E(\sqrt{1-l_m^2}) + 2^{m-1}\,l_m\,c_m^2\,K(k) \qquad \text{(wegen (34))},$$

und

$$\sum_{m=2}^{n}\left\{c_{m-1}\,E(\sqrt{1-l_{m-1}^2}) - c_m\,E(\sqrt{1-l_m^2})\right\} = K(k)\sum_{m=2}^{n} 2^{m-1}\,l_m\,c_m^2 = K(k)\,.\,A_n$$

w. z. b. w.

Zusatz: *Für $m \geqq 3$ ist, wegen (30) und (31)*

$$\left.\begin{aligned} l_m\,c_m^2 &= \frac{c_{m-1}-d_{m-1}}{c_{m-1}+d_{m-1}}\,.\,c_m^2 = \frac{c_{m-1}^2-d_{m-1}^2}{4} = \frac{(c_{m-2}+d_{m-2})^2-4\,c_{m-2}\,.\,d_{m-2}}{16} = \\ &= \left(\frac{c_{m-2}-d_{m-2}}{4}\right)^2 = \left(\frac{c_{m-3}+d_{m-3}-2\sqrt{c_{m-3}\,.\,d_{m-3}}}{8}\right)^2 = \frac{(\sqrt{c_{m-3}}-\sqrt{d_{m-3}})^4}{64} \\ l_2\,c_2^2 &= \tfrac{1}{4}\cos^2\alpha. \end{aligned}\right\} \quad (45)$$

Man findet somit, wegen (30), (32) und (45) [1])

$$\left.\begin{aligned} A_2 &= \sum_{m=2}^{2} 2^{m-1}\,l_m\,c_m^2 = \tfrac{1}{2}\cos^2\alpha; \\ A_3 &= \sum_{m=2}^{3} 2^{m-1}\,l_m\,c_m^2 = \tfrac{1}{2}\cos^2\alpha + \tfrac{1}{4}(1-\sin\alpha)^2 = \tfrac{1}{4}(1-\sin\alpha)(3+\sin\alpha); \\ A_4 &= \sum_{m=2}^{4} 2^{m-1}\,l_m\,c_m^2 = \tfrac{1}{2}\cos^2\alpha + \tfrac{1}{4}(1-\sin\alpha)^2 + \tfrac{1}{8}(1-\sqrt{\sin\alpha})^4; \\ A_5 &= \sum_{m=2}^{5} 2^{m-1}\,l_m\,c_m^2 = \tfrac{1}{2}\cos^2\alpha + \tfrac{1}{4}(1-\sin\alpha)^2 + \\ &\quad + \tfrac{1}{8}(1-\sqrt{\sin a})^4 + \tfrac{1}{4}\left\{\cos\left(\frac{\pi}{4}-\frac{\alpha}{2}\right) - \sqrt[4]{\sin\alpha}\right\}^4 \end{aligned}\right\} \quad (46)$$

u. s. w.

Anwendung von (26) und (35) auf (44) liefert unmittelbar

Satz VII: *Für ganzes $n \geqq 1$ ist*

$$\left.\begin{aligned} E(k_1) &= c_n\,F(-\tfrac{1}{2}, -\tfrac{1}{2}; 1; l_n^2) + \frac{c_n}{2}\,l_n^2\,\log\frac{4}{l_n}\,.\,F(\tfrac{1}{2}, \tfrac{3}{2}; 2; l_n^2) \\ &- \frac{2c_n}{\pi}\sum_{m=0}^{\infty}\frac{1}{(2m+1)(2m+2)}\sum_{q=m}^{\infty}\frac{\Gamma(q+\frac{1}{2})\,.\,\Gamma(q+\frac{3}{2})}{q!\,(q+1)!}\,l_n^{2q+2} + A_n\,K(k_1), \end{aligned}\right\} \quad (47)$$

wo $K(k_1)$ durch (35), und A_n durch (46) bestimmt ist.

---

[1]) Vgl. den Zusatz am Ende dieses Berichtes. Weiter ist $A_n = \sum_{m=1}^{n-1} 2^{m-2}\,l_m^2\,c_m^2$.

Wir lassen die explizite Darstellung der Sonderfälle $n = 1, \ldots, 5$ fort, und geben nur die Hauptglieder für $n = 3$, 4 und 5. Sie lauten [1])

$$E(k_1) \approx c_n + A_n K^{*\prime}(k_1),$$

also, wegen (30) und (46):

$$E(k_1) \approx \left(\frac{1+\sqrt{\sin\alpha}}{2}\right)^2 + \frac{1}{2}\frac{(1-\sin\alpha)(3+\sin\alpha)}{(1+\sqrt{\sin\alpha})^2}\log 2\left(\frac{1+\sqrt{\sin\alpha}}{1-\sqrt{\sin\alpha}}\right); \quad (E^* III')$$

$$E(k_1) \approx \left(\frac{\cos\left(\frac{\pi}{4}-\frac{\alpha}{2}\right)+\sqrt[4]{\sin\alpha}}{2}\right)^2 +$$

$$+ \frac{\left\{\frac{1}{2}\cos^2\alpha+\frac{1}{4}(1-\sin\alpha)^2+\frac{1}{8}(1-\sqrt{\sin\alpha})^4\right\}}{\left(\cos\left(\frac{\pi}{4}-\frac{\alpha}{2}\right)+\sqrt[4]{\sin\alpha}\right)^2}\log 2\left(\frac{\cos\left(\frac{\pi}{4}-\frac{\alpha}{2}\right)+\sqrt[4]{\sin\alpha}}{\cos\left(\frac{\pi}{4}-\frac{\alpha}{2}\right)-\sqrt[4]{\sin\alpha}}\right); \quad (E^* IV')$$

$$E(k_1) \approx \left(\frac{1+\sqrt{\sin\alpha}+2\sqrt{\cos\left(\frac{\pi}{4}-\frac{\alpha}{2}\right)\cdot\sqrt[8]{\sin\alpha}}}{2}\right)^2 +$$

$$+ \frac{\left[\frac{1}{2}\cos^2\alpha+\frac{1}{4}(1-\sin\alpha)+\frac{1}{8}(1-\sqrt{\sin\alpha})^4+\frac{1}{4}\left\{\cos\left(\frac{\pi}{4}-\frac{\alpha}{2}\right)-\sqrt[4]{\sin\alpha}\right\}^4\right]}{\left(1+\sqrt{\sin\alpha}+2\sqrt{\cos\left(\frac{\pi}{4}-\frac{\alpha}{2}\right)\cdot\sqrt[8]{\sin\alpha}}\right)^2}\cdot$$

$$\log 2\left(\frac{1+\sqrt{\sin\alpha}+2\sqrt{\cos\left(\frac{\pi}{4}-\frac{\alpha}{2}\right)\cdot\sqrt[8]{\sin\alpha}}}{1+\sqrt{\sin\alpha}-2\sqrt{\cos\left(\frac{\pi}{4}-\frac{\alpha}{2}\right)\cdot\sqrt[8]{\sin\alpha}}}\right) \quad (E^* V')$$

Die Fehler sind wegen (47)

$$\approx l_n^2\left(\frac{c_n}{4}+\frac{c_n}{2}\log\frac{4}{l_n}-\frac{c_n}{2}+\frac{A_n}{2^{n+1}\cdot c_n}\log\frac{4}{l_n}-\frac{A_n}{2^{n+1}c_n}\right)=$$

$$= \frac{c_n l_n^2}{4}\left(2\log\frac{4}{l_n}-1\right)+\frac{A_n l_n^2}{2^{n+1}c_n}\left(\log\frac{4}{l_n}-1\right).$$

Sie betragen im Gebiet $\frac{\pi}{4} \leqq \alpha \leqq \frac{\pi}{2}$

n	Fehler bei Verwendung von $E^{*\prime}$
3	$\leqq 1{,}9 \cdot 10^{-5}$
4	$6{,}4 \cdot 10^{-11}$
5	$7{,}6 \cdot 10^{-21}$

[1]) Vgl. den Zusatz.

Die Anwendung der Annäherungsformeln E' im Gebiet $0 \leqq \alpha \leqq \frac{\pi}{4}$, und $E^{*\prime}$ im Gebiet $\frac{\pi}{4} \leqq \alpha \leqq \frac{\pi}{2}$ verbürgt somit schon Genauigkeit

in 4 Stellen für $q = 3$,
„ 9 „ „ $q = 4$,
„ 19 „ „ $q = 5$,

im ganzen Gebiet $0 \leqq \alpha \leqq \frac{\pi}{2}$.

(Zusatz bei der Korrektur):

Aus

$$1 + l_m = \frac{c_{m-1}}{c_m}, \text{ und } l_{m-1} = \frac{2\sqrt{l_m}}{1 + l_m} \quad \text{(vgl. (33))}$$

ergibt sich

$$l_{m-1}\, c_{m-1} = 2\, c_m \sqrt{l_m}\,.$$

Also ist für $m > 1$

$$2^{2m-2} \cdot \frac{c_m^2}{c_1^2} = \prod_{p=2}^{m} \frac{2^2\, c_p^2}{c_{p-1}^2} = \prod_{p=2}^{m} \frac{l_{p-1}^2}{l_p} = \frac{l_1}{l_m} \cdot (l_1 \cdot l_2 \ldots . l_{m-1}),$$

oder

$$A_n = \sum_{m=2}^{n} 2^{m-1}\, l_m\, c_m^2 = l_1 \sum_{m=2}^{n} \frac{l_1\, l_2 \ldots . l_{m-1}}{2^{m-1}};\ (n > 1) \quad . \quad . \quad . \quad (\alpha)$$

(vgl. (46) und (47)).

Unter Vernachlässigung von l_n^2 u.s.w. ergibt sich wegen (α) und (35) für die Hauptglieder von (47) den einfach gebildeten Ausdruck

$$\left.\begin{aligned} E(k_1) &\approx c_n + K(k_1) \sum_{m=2}^{n} 2^{m-1}\, l_m\, c_m^2 + \frac{c_n}{2}\, l_n^2\, log\, \frac{4}{l_n} \\ &\approx c_n + \frac{1}{2^{n-1}\, c_n}\, log\, \frac{4}{l_n} \sum_{m=2}^{n+1} 2^{m-1}\, c_m^2 = \\ &= c_n + \frac{l_1}{2^{n-1}\, c_n} \sum_{m=1}^{n} \frac{l_1 \cdot l_2 \ldots . l_m}{2^m}. \end{aligned}\right\} \quad . \quad . \quad (\beta)$$

(Fortsetzung folgt.)

Mathematics. — *Ueber die Entwicklung der unvollständigen elliptischen Integrale erster und zweiter Art in stark konvergenten Reihen.* III. Von S. C. VAN VEEN. (Communicated by Prof. J. G. VAN DER CORPUT.)

(Communicated at the meeting of November 29, 1941.)

§ 2. *α in der Nähe von $\frac{\pi}{2}$; β klein.*

A. Das unvollständige elliptische Integral erster Art.

$$F(\sin\alpha,\beta)=\int_0^{\beta}\frac{d\varphi}{\sqrt{1-\sin^2\alpha\,.\,\sin^2\varphi}}=\int_0^{\beta}\frac{\cos\varphi\,.\,d\varphi}{\sqrt{(1-\sin^2\varphi)\,(1-\sin^2\alpha\sin^2\varphi)}}=$$

$$=\int_0^{\beta}\frac{\cos\varphi\,d\varphi}{\sqrt{(1-\sin\alpha\,.\,\sin^2\varphi)^2-(1-\sin\alpha)^2\sin^2\varphi}}$$

$$=\int_0^{\beta}\frac{\cos\varphi\,d\varphi}{(1-\sin\alpha\,.\,\sin^2\varphi)}\sum_{n=0}^{\infty}\left(\frac{1.3.5\ldots(2n-1)}{2.4.6\ldots2n}\right)\frac{(1-\sin\alpha)^{2n}\sin^{2n}\varphi}{(1-\sin\alpha\sin^2\varphi)^{2n}}= \tag{25}$$

$$=\sum_{n=0}^{\infty}\left(\frac{1.3.5\ldots(2n-1)}{2.4.6\ldots2n}\right)(1-\sin\alpha)^{2n}\int_0^{\sin\beta}\frac{x^{2n}\,dx}{(1-\sin\alpha\,.\,x^2)^{2n+1}}=$$

$$=\sum_{n=0}^{\infty}\left(\frac{1.3.5\ldots(2n-1)}{2.4.6\ldots2n}\right)\frac{(1-\sin\alpha)^{2n}}{(\sin\alpha)^{n+\frac{1}{2}}}\int_0^{\sin\beta\sqrt{\sin\alpha}}\frac{y^{2n}\,dy}{(1-y^2)^{2n+1}}$$

Setzt man

$$y=\frac{e^{2z}-1}{e^{2z}+1},\quad\text{oder}\quad z=\tfrac{1}{2}\log\frac{1+y}{1-y},$$

also

$$\frac{dz}{dy}=\frac{1}{1-y^2};\quad\frac{y}{1-y^2}=\frac{e^{4z}-1}{4e^{2z}}=\frac{e^{2z}-e^{-2z}}{4}$$

und

$$\frac{1-\sin\alpha}{\sqrt{\sin\alpha}}=x;\quad\frac{1+\sin\beta\sqrt{\sin\alpha}}{1-\sin\beta\sqrt{\sin\alpha}}=w,$$

so wird

$$\left.\begin{aligned}\int_0^{\sin\beta\sqrt{\sin\alpha}} \frac{y^{2n}\,dy}{(1-y^2)^{2n+1}} &= \int_0^{\frac{1}{2}\log w}\left(\frac{e^{2z}-e^{-2z}}{4}\right)^{2n} dz = \frac{1}{4^{2n}}\sum_{p=0}^{2n}(-1)^p\binom{2n}{p}\int_0^{\frac{1}{2}\log w} e^{4(n-p)z}\,dz = \\ &= \frac{1}{4^{2n+1}}\sum_{\substack{p=0\\p\neq n}}^{2n}(-1)^p\binom{2n}{p}\frac{w^{2(n-p)}}{n-p} + \frac{(-1)^n}{4^{2n}}\binom{2n}{n}\tfrac{1}{2}\log w = \\ &= \frac{(-1)^n}{2^{2n+1}}\left(\frac{1.3.5\ldots(2n-1)}{2.4.6\ldots 2n}\right)\log w + \frac{w^{2n}}{4^{2n+1}}\sum_{\substack{p=0\\p\neq n}}^{2n}(-1)^p\binom{2n}{p}\frac{w^{-2p}}{n-p}\end{aligned}\right\} \quad (26)$$

Aus (25) und (26) ergibt sich

$$\left.\begin{aligned}F(\sin\alpha,\beta) &= \frac{1}{2\sqrt{\sin\alpha}}\log w\sum_{n=0}^{\infty}(-1)^n\left(\frac{1.3.5\ldots(2n-1)}{2.4.6\ldots 2n}\right)^2\left(\frac{x}{2}\right)^{2n} \\ &+ \frac{1}{4\sqrt{\sin\alpha}}\sum_{n=1}^{\infty}\left(\frac{1.3.5\ldots(2n-1)}{2.4.6\ldots 2n}\right)\left(\frac{xw}{4}\right)^{2n}\sum_{\substack{p=0\\p\neq n}}^{2n}(-1)^p\binom{2n}{p}\frac{w^{-2p}}{n-p},\end{aligned}\right\} \quad (27)$$

mit $$x = \frac{1-\sin\alpha}{\sqrt{\sin\alpha}};\quad w = \frac{1+\sin\beta\sqrt{\sin\alpha}}{1-\sin\beta\sqrt{\sin\alpha}}$$

Die erste Reihe in (27) konvergiert für

$$\frac{x}{2} = \frac{1-\sin\alpha}{2\sqrt{\sin\alpha}} \leqq 1, \quad \text{oder} \quad \sin\alpha \geqq 3-2\sqrt{2};\quad \alpha \geqq 9^\circ.53'.$$

Die Konvergenz ist sehr stark, wenn α in der Nähe von $\frac{\pi}{2}$ liegt. Auch die zweite Reihe in (27) konvergiert dann stark, *wenn wenigstens sin β nicht in der Nähe von 1 liegt.* Wegen

$$\left|\sum_{\substack{p=0\\p\neq n}}^{2n}(-1)^p\binom{2n}{p}\frac{w^{-2p}}{n-p}\right| < \sum_{p=0}^{2n}\binom{2n}{p}w^{-2p} = (1+w^{-2})^{2n} = \left\{\frac{2(1+\sin^2\beta\sin\alpha)}{(1+\sin\beta\sqrt{\sin\alpha})^2}\right\}^{2n}$$

bildet

$$\frac{xw}{4}\cdot\frac{2(1+\sin^2\beta\sin\alpha)}{(1+\sin\beta\sqrt{\sin\alpha})^2} = \frac{x}{2}\,\frac{1+\sin^2\beta\sin\alpha}{1-\sin^2\beta\sin\alpha} < 1$$

oder

$$0 \leqq \sin^2\beta < \frac{2-x}{(2+x)\sin\alpha} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (28)$$

eine *hinreichende* Bedingung für die Konvergenz der zweiten Reihe in (27). (Die erste Reihe konvergiert dann auch, wegen $x \leqq 2$) (vgl. Fig. Proc. Ned. Akad. v. Wetensch., Amsterdam, **44** 911 (1941)).

Das Hauptglied der Reihe (27) ist

$$F(\sin\alpha,\beta) \approx \frac{1}{2\sqrt{\sin\alpha}}\log w = \frac{1}{2\sqrt{\sin\alpha}}\log\frac{1+\sin\beta\sqrt{\sin\alpha}}{1-\sin\beta\sqrt{\sin\alpha}}, \quad (29)$$

B. Das unvollständige elliptische Integral zweiter Art.

$$
\left.
\begin{aligned}
E(\sin\alpha,\beta) &= \int_0^{\beta} \sqrt{1-\sin^2\alpha\,\sin^2\varphi}\,d\varphi = \int_0^{\beta} \frac{\cos\varphi\,(1-\sin^2\alpha\,\sin^2\varphi)}{\sqrt{(1-\sin^2\varphi)\,(1-\sin^2\alpha\,\sin^2\varphi)}}\,d\varphi = \\
&= \int_0^{\beta} \frac{\cos\varphi\,(1-\sin^2\alpha\,.\,\sin^2\varphi)\,d\varphi}{\sqrt{(1-\sin\alpha\,.\,\sin^2\varphi)^2-(1-\sin\alpha)^2\,\sin^2\varphi}} = \\
&= \int_0^{\beta} \frac{\cos\varphi\,(1-\sin^2\alpha\,.\,\sin^2\varphi)\,d\varphi}{1-\sin\alpha\,.\,\sin^2\varphi} \sum_{n=0}^{\infty} \left(\frac{1.3.5\ldots(2n-1)}{2.4.6\ldots 2n}\right) \frac{(1-\sin\alpha)^{2n}\,\sin^{2n}\varphi}{(1-\sin\alpha\,.\,\sin^2\varphi)^{2n}} \\
&= \frac{1}{\sqrt{\sin\alpha}} \sum_{n=0}^{\infty} \left(\frac{1.3.5\ldots(2n-1)}{2.4.6\ldots 2n}\right) x^{2n} \int_0^{\sin\beta\sqrt{\sin\alpha}} \frac{y^{2n}\,(1-\sin\alpha\,y^2)}{(1-y^2)^{2n+1}}\,dy = \\
&= \frac{1}{\sqrt{\sin\alpha}} \sum_{n=0}^{\infty} \left(\frac{1.3.5\ldots(2n-1)}{2.4.6\ldots 2n}\right) x^{2n} \int_0^{\frac{\log w}{2}} \left(\frac{e^{2z}-e^{-2z}}{4}\right)^{2n} \left\{1-\sin\alpha\,.\left(\frac{e^z-e^{-z}}{e^z+e^{-z}}\right)^2\right\} dz.
\end{aligned}
\right\} \quad (30)
$$

Für $n \geqq 1$ ist

$$
\left.
\begin{aligned}
&\int_0^{\frac{\log w}{2}} \left(\frac{e^{2z}-e^{-2z}}{4}\right)^{2n} \left\{1-\sin\alpha\left(\frac{e^z-e^{-z}}{e^z+e^{-z}}\right)^2\right\} dz = \\
&= \frac{1}{4^{2n}} \int_0^{\frac{\log w}{2}} (e^{2z}-e^{-2z})^{2n}\,dz - \frac{\sin\alpha}{4^{2n}} \int_0^{\frac{\log w}{2}} \frac{(e^{2z}-e^{-2z})^{2n}\,(e^z-e^{-z})^4}{(e^{2z}-e^{-2z})^2}\,dz = \\
&= \frac{1}{4^{2n}} \int_0^{\frac{\log w}{2}} (e^{2z}-e^{-2z})^{2n}\,dz - \frac{\sin\alpha}{4^{2n}} \int_0^{\frac{\log w}{2}} (e^{2z}-e^{-2z})^{2n}\,dz - \frac{8\sin\alpha}{4^{2n}} \int_0^{\frac{\log w}{2}} (e^{2z}-e^{-2z})^{2n-2}\,dz \\
&+ \frac{\sin\alpha}{4^{2n-1}} \int_0^{\frac{\log w}{2}} (e^{2z}-e^{-2z})^{2n-z}\,(e^{2z}+e^{-2z})\,dz = \\
&= \frac{1-\sin\alpha}{4^{2n+1}} \sum_{\substack{p=0 \\ p\neq n}}^{2n} (-1)^p \binom{2n}{p} \frac{w^{2(n-p)}}{n-p} + \frac{(-1)^n}{4^{2n}} \binom{2n}{n} \frac{(1-\sin\alpha)}{2} \log w \\
&- \frac{8\sin\alpha}{4^{2n+1}} \sum_{\substack{p=0 \\ p\neq n-1}}^{2n-2} (-1)^p \binom{2n-2}{p} \frac{w^{2(n-1-p)}}{n-1-p} - \\
&\qquad \frac{(-1)^{n-1}.8}{4^{2n}} \cdot \binom{2n-2}{n-1} \frac{\sin\alpha}{2} \cdot \log w + \frac{\sin\alpha}{4^{2n-\frac{1}{2}}} \frac{(w-w^{-1})^{2n-1}}{2n-1}.
\end{aligned}
\right\} \quad (31)
$$

Für $n = 0$ ist

$$\int_0^{\frac{\log w}{2}} \left\{1-\sin\alpha\left(\frac{e^z-e^{-z}}{e^z+e^{-z}}\right)^2\right\} dz = \int_0^{\frac{\log w}{2}} \left\{(1-\sin\alpha)+\frac{4\sin\alpha\,.\,e^{2z}}{(e^{2z}+1)^2}\right\} dz = \left.\right\} \quad (32)$$

$$= \frac{1-\sin\alpha}{2}\log w - \frac{2\sin\alpha}{w+1} + \sin\alpha = \frac{1-\sin\alpha}{2}\log w + \sin\alpha\left(\frac{w-1}{w+1}\right). \left.\right\}$$

Aus (30), (31) und (32) ergibt sich nach einiger Reduktion die Entwicklung

$$E(\sin\alpha,\beta) = \frac{x}{2}\log w + \sin\beta\,.\,\sin\alpha + \frac{2w\sqrt{\sin\alpha}}{w^2-1}\left[1-\sqrt{1-\left\{\frac{x(w^2-1)}{4w}\right\}^2}\right] \left.\right\}$$

$$+ \frac{1}{2\sqrt{\sin\alpha}}\log w \sum_{n=1}^{\infty}(-1)^n\left(\frac{1.3.5\ldots(2n-1)}{2.4.6\ldots 2n}\right)^2\left(\frac{x}{2}\right)^{2n}\left(1+\frac{2n+1}{2n-1}\sin\alpha\right) \left.\right\} \quad (33)$$

$$+ \frac{1}{4\sqrt{\sin\alpha}}\sum_{n=1}^{\infty}\left(\frac{1.3.5\ldots(2n-1)}{2.4.6\ldots 2n}\right)\left(\frac{xw}{4}\right)^{2n} \left.\right\}$$

$$\left\{(1-\sin\alpha)\sum_{\substack{p=0\\p\neq n}}^{2n}(-1)^p\binom{2n}{p}\frac{w^{-2p}}{n-p} - 8\sin\alpha\sum_{\substack{p=0\\p\neq n-1}}^{2n-2}(-1)^p\binom{2n-2}{p}\cdot\frac{w^{-2p-2}}{n-p-1}\right\},$$

mit

$$x = \frac{1-\sin\alpha}{\sqrt{\sin\alpha}};\quad w = \frac{1+\sin\beta\sqrt{\sin\alpha}}{1-\sin\beta\sqrt{\sin\alpha}}$$

mit dem Hauptglied

$$E(\sin\alpha,\beta) \approx \frac{x}{2}\log w + \sin\beta\,.\,\sin\alpha = \left.\right\}$$

$$= \frac{1-\sin\alpha}{2\sqrt{\sin\alpha}}\,.\,\log\frac{1+\sin\beta\,.\,\sqrt{\sin\alpha}}{1-\sin\beta\,.\,\sqrt{\sin\alpha}} + \sin\beta\,.\,\sin\alpha \left.\right\} \quad (34)$$

Die Konvergenzbedingungen der Reihen (33) sind identisch mit den Bedingungen der Reihen (27), also:

Hinreichende Bedingung für die Konvergenz der Reihe (33) ist

$$0 \leqq \sin^2\beta < \frac{2-x}{(2+x)\sin\alpha} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (35)$$

(Fortsetzung folgt.)

Mathematics. — *Domaines d'univalence et d'étoilement des fonctions holomorphes à partie réelle positive dans un demi-plan.* Par Prof. J. WOLFF. (Communicated by Prof. J. G. VAN DER CORPUT.)

(Communicated at the meeting of November 29, 1941.)

On sait que toute fonction $w(z)$ holomorphe et à partie réelle positive dans le demi-plan $D(x>0)$ de la variable complexe $z=x+iy$ peut être représentée d'une seule manière par la formule

$$w(z)=\lambda z+H(z) \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (1)$$

où la constante λ, appelée la dérivée angulaire de $w(z)$ á l'infini, est un nombre fini positif ou nul et $H(z)$ une fonction holomorphe à partie réelle positive dans D, dont la dérivée angulaire à l'infini est nulle.

Il est connu que $zH(z)$ a pour z infini une limite angulaire μ positive ou infinie, et que, $u(z)$ étant la partie réelle de $H(z)$, sur toute droite $y=$ const. le produit $x.u(x+iy)$ ne décroît jamais lorsque x croît et tend vers μ pour $x\to\infty$:

$$u(z)\leqq\frac{\mu}{x},\quad \lim_{x\to\infty} x.u(x+iy)=\mu \quad . \quad . \quad . \quad . \quad . \quad (2)$$

Considérons dans D la classe $C(\lambda,\mu)$ des fonctions (1) pour lesquelles les nombres λ et μ sont des constantes données, $\lambda>0$, $0<\mu<\infty$.

Posons les deux questions:

1. Chercher la borne inférieure b des nombres reéls et non négatifs a tels que toutes les fonctions de la classe $C(\lambda,\mu)$ soient univalentes dans les demi-plans $x>a$.

2. Chercher la borne inférieure b' des nombres a' tels que pour toutes les fonctions (1) de $C(\lambda,\mu)$ les images des droites verticales x const. $>a'$, réalisées par la transformation $w=w(z)$, coupent toute droite horizontale $y=$ const. en un seul point au plus.

Le demi-plan $\triangle(x>b')$ est le plus grand demi-plan dans D où toute fonction $w(z)$ de $C(\lambda,\mu)$ est *étoilée par rapport au point à l'infini de l'axe réel.* Il est clair que $\triangle(x>b')$ est le plus grand demi-plan dans D, où toute fonction de $C(\lambda,\mu)$ jouit de la propriété:

$$\text{partie réelle de la dérivée}=\Re(w')>0. \quad . \quad . \quad . \quad . \quad . \quad (3)$$

L'étoilement entraîne l'univalence, donc $b\leqq b'$.

On voit l'analogie avec les recherches connues sur le plus grand cercle $|z|=\varrho$ d'univalence et d'étoilement pour les fonctions $w=az+\ldots,\ |a|<1$, holomorphes et de valeurs absolues <1 dans le cercle-unité [1]. L'analogie se montre mieux encore, si nous supposons $\lambda>1$, ce qui ne nuit pas à la généralité; car alors le point à l'infini est le point attractif pour les itérés $z_1=w(z)$, $z_2=w(z_1),\ldots$ et joue le rôle du point attractif $z=0$ dans le cercle-unité, l'inégalité $\Re(w)>x$ entrant à la place de l'inégalité $|w|<|z|$ dans le cercle-unité.

L'inégalité connue $|H'(z)|\leqq\frac{u}{x}$ et l'inégalité (2) montrent que

$$|H'(z)|\leqq\frac{\mu}{x^2}. \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (4)$$

---

[1] Par exemple: J. DIEUDONNÉ, *Polynomes et fonctions bornées d'une variable complexe.* Ann. Sc. de l'École normale supérieure, série 3, tome **48**, 1931, p. 307–358.

De (1) et (4) on tire

$$\Re(w') \geqq \lambda - \frac{\mu}{x^2}. \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (5)$$

De (5) on conclut immédiatement que $b' \leqq \sqrt{\frac{\mu}{\lambda}}$.

Cherchons maintenant les fonctions de la classe $C(\lambda, \mu)$ pour lesquelles $\triangle\left(x > \sqrt{\frac{\mu}{\lambda}}\right)$ est le plus grand demi-plan d'étoilement. Dans ce but réprésentons $H(z)$ par une intégrale de STIELTJES étendue sur l'axe imaginaire $z = t\,i$, $-\infty < t < \infty$:

$$H(z) = \frac{1}{\pi}\int_{t=-\infty}^{\infty}\left(\frac{1}{z-ti} + \frac{1}{1+ti}\right) d\psi(t) + c \quad . \quad . \quad . \quad . \quad (6)$$

où $\psi(t)$ est une fonction non-décroissante telle que les intégrales $\int_{1}^{\infty}\frac{d\psi}{t^2}$ et $\int_{-\infty}^{-1}\frac{d\psi}{t^2}$ sont convergentes, et c une constante. De (6) résulte

$$u(z) = \frac{1}{\pi}\int_{t=-\infty}^{\infty}\frac{x\,d\psi}{|z-ti|^2} + \frac{1}{\pi}\int_{t=-\infty}^{\infty}\frac{d\psi}{|1+ti|^2} + \Re(c). \quad . \quad . \quad (7)$$

La somme des deux derniers termes dans (7) étant $\geqq 0$, (2) exige qu'elle soit $= 0$:

$$u(z) = \frac{x}{\pi}\int_{t=-\infty}^{\infty}\frac{d\psi}{|z-ti|^2}. \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (8)$$

Il suit de (2) et de (8) que

$$\lim_{x\to\infty}\int_{t=-\infty}^{\infty}\frac{x^2}{x^2+t^2}\,d\psi = \pi\mu. \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (9)$$

Soit N un nombre fini positif. En vertu de (2) et (9) on a pour $z = x > 0$

$$\int_{t=-N}^{N}\frac{x^2}{x^2+t^2}\,d\psi \leqq \pi\mu.$$

Remplaçons le premier membre de la dernière inégalité par sa limite pour $x \to \infty$. Il vient

$$\int_{t=-N}^{N} d\psi \leqq \pi\mu.$$

Cela étant quel que soit N, nous concluons que

$$\int_{t=-\infty}^{\infty} d\psi \leqq \pi\mu.$$

D'autre part (9) montre que $\int_{t=-\infty}^{\infty} d\psi \geqq \pi\mu$, donc

$$\int_{t=-\infty}^{\infty} d\psi = \pi\mu. \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (10)$$

Étudions maintenant la conduite de $u(z)$ dans un demi-plan $\triangle_0$ $(x > x_0 > 0)$. Soit ε un nombre positif. L'équation (10) met en évidence l'existence d'un nombre positif N tel que

$$\int_{|t|>N} d\psi < \varepsilon \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (11)$$

z étant dans $\triangle_0$ on déduit de (11)

$$\frac{x}{\pi} \int_{|t|>N} \frac{d\psi}{|z-ti|^2} \leqq \frac{\varepsilon}{\pi x} \leqq \frac{\varepsilon}{\pi x_0} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (12)$$

Pour $z \to \infty$ dans $\triangle_0$ nous avons en vertu de (10)

$$\frac{x}{\pi} \int_{|t| \leqq N} \frac{d\psi}{|z-ti|^2} \to 0, \text{ } \textit{uniformément dans } \triangle_0. \quad . \quad . \quad . \quad . \quad . \quad (13)$$

(8), (12) et (13) montrent que

$$u(z) \to 0 \text{ pour } z \to \infty, \text{ } \textit{uniformément dans } \triangle_0. \quad . \quad . \quad . \quad . \quad (14)$$

Et, parceque $|H'(z)| \leqq \frac{u}{x}$ et $\Re(w') = \lambda + \Re(H')$, il vient

$$\lim_{|z| \to \infty} \Re(w') = \lambda > 0, \text{ } \textit{uniformément dans } \triangle_0. \quad . \quad . \quad . \quad . \quad (15)$$

Conséquence de (15) est: condition nécessaire et suffisante pour que $\triangle\left(x > \left|\frac{\mu}{\lambda}\right.\right)$ soit le plus grand demi-plan d'étoilement est que la droite $x = \sqrt{\frac{\mu}{\lambda}} = x_0$ contienne un point *fini* $z_0 = x_0 + iy_0$ satisfaisant à $\Re\{w'(z_0)\} = 0$, donc $\Re\{H'(z_0)\} = -\lambda$. Or, en vertu de (6)

$$H'(z_0) = -\frac{1}{\pi} \int_{t=-\infty}^{\infty} \frac{d\psi}{(z_0-ti)^2},$$

d'où en vertu de (10)

$$\Re\{H'(z_0)\} \geqq -\frac{1}{\pi} \int_{t=-\infty}^{\infty} \frac{d\psi}{|z_0-ti|^2} \geqq -\frac{\mu}{x_0^2} = -\lambda$$

et l'égalité $\Re\{H'(z_0)\} = -\lambda$ ne se produit que si $\psi(t)$ est constante pour $y > y_0$ et pour $y < y_0$, donc si $\psi(y_0+0) - \psi(y_0-0) = \pi\mu$.

Les fonctions cherchées satisfont par conséquent à

$$H'(z) = -\frac{\mu}{(z-iy_0)^2}$$

dans D, donc

$$H(z) = \frac{\mu}{z-iy_0}, \quad -\infty < y_0 < \infty.$$

La constante additive est nulle en vertu de $x\,H(x) \to \mu$ pour $x \to \infty$. Inversement pour toutes ces fonctions $H'(z_0) = -\frac{\mu}{x_0^2} = -\lambda$, donc $w'(z_0) = 0$, par suite $\triangle\left(x > \sqrt{\frac{\mu}{\lambda}}\right)$ est le plus grand demi-plan d'univalence et d'étoilement pour les fonctions $w(z) = \lambda z + \frac{\mu}{z-iy_0}$ et pour aucune autre fonction de la classe $C(\lambda, \mu)$. Nous avons démontré le

THÉORÈME I. Toute fonction de la classe $C(\lambda, \mu)$ est étoilée, donc univalente dans $\triangle\left(x > \sqrt{\frac{\mu}{\lambda}}\right)$. Les seules fonctions de la classe pour lesquelles $\triangle$ est le plus grand demi-plan d'univalence, donc d'étoilement, sont

$$w(z) = \lambda z + \frac{\mu}{z-ki}, \quad -\infty < k < \infty.$$

L'équation (14) donne le

THÉORÈME II. Toute fonction $H(z)$ holomorphe et à partie réelle positive dans D, pour laquelle la limite angulaire de $z\,H(z)$ pour z infini est finie, jouit de la propriété que dans tout demi-plan $x > x_0 > 0$ la partie réelle de $H(z)$, donc de même la dérivée $H'(z)$, tendent uniformément vers zéro quand $z \to \infty$.

Botany. — *The growth of the petioles of waterplants in solutions of phytohormones.* (From the Botanical Institute, Government University, Leyden.) II. By G. L. FUNKE. (Communicated by Prof. L. G. M. BAAS BECKING.)

(Communicated at the meeting of November 29, 1941.)

V. *Cell elongation—cell division.*

I have measured the length of the cells of epidermis and subepidermal layer in more than 150 petioles and in each at 5 places: above, at $\frac{1}{4}$ from the top, in the middle, at $\frac{3}{4}$ from the top and below. They confirm completely the outcome of the thousands of measurements made in foregoing years, viz. that in the vast majority of cases the definite length is reached partly by cell elongation and partly by the formation of new cells. It is impossible to say more or less exactly in how far each of them accounts for the rapid increase in length of the petiole; now one may take the upperhand, now the other; sometimes elongation or division is more important in the whole extension of the petiole, sometimes only in one or some parts of it. Many factors probably influence both processes of growth, many of which, esp. the internal ones, escape thus far our control (age, respiration, assimilation, food storage, etc.). There is hardly any phenomenon in plant life so capricious as the cell dimensions. Yet there is one rule which holds true fairly generally, viz. that cell elongation is the principle means of reaching the definite length whenever a leaf is in unfavorable conditions, internal or external. Old age, sickness, poisonous solutions, etc. are invariably followed by strong cell elongation, and reversely, whenever I find abnormally long cells I feel intitled to conclude from that fact that the petiole in question has somehow suffered from injury (comp. 2). A striking example is shown by *Limnanthemum nymphaeoides* (9); concentrations 0.25 mg/L and 5 mg/L alpha-naphtalene acetic acid are resp. sub- and super-optimal for the longitudinal growth of its petioles; although the lengths reached in these concentrations are about the same, the cell lengths in 5 mg/L are abnormal for this species and $\pm$ 2 à 3 times those in 0.25; externally no difference is to be seen, but all measurements show that the growth had been achieved in different circumstances and therefore in totally different ways.

A selection of my measurements may illustrate sufficiently what has been said above.

Table 6*a* gives the data of a few specimens of *N. amazonum*. It happens that with the progression of the season the petioles show longer cells; therefore control specimens have been measured at different periods in summer. I would point out the following facts: No. 2 in 0.25 mg/L naphtalene acetic acid, although the longest petiole of Nos. 2—5, has by far the shortest cells; the concentrations 0.5 and 1 mg/L are obviously noxious. No. 4 has much longer cells than No. 3, although it is shorter; this is an example of the capriciousness which one meets so often and for which there is not always a ready explanation. No. 7, three times longer than No. 6, has cells of the same order of length; Nos. 8 and 9 have about twice the length of No. 7; if we only compared these three, we would be inclined to assume cell elongation as the only means of reaching the greater length; this assumption, however, has to be abandoned when we compare Nos. 8 and 9 with No. 6. No. 11 shows long cells over the whole extension, but No. 10, of the same age, is only $\pm$ $\frac{1}{5}$ its length and this relation is nowhere to be found in the cell dimensions. No. 14 is much shorter than No. 13; it is a striking example of a petiole which has done its utmost without succeeding in reaching the surface of the water; this exertion is consequently clearly expressed by its abnormally long cells.

Table 6*b*, *N. Boucheana*. No. 3 was a rather old leaf which was visibly injured by the 0.5 mg/L solution; it was strongly twisted and broke off while being measured some days

TABLE 6*a*. Celllengths of Nymphaea amazonum.

No.	Particulars (conc. of horm. mg/L; depth water; date)	Length pet. in cm	Dimensions leaf blade in mm (middle vein — greatest breadth)	Average length of cells in micra									
				Epidermis					Subepidermis				
				a	$\frac{1}{4}$	*m*	$\frac{3}{4}$	*b*	*a*	$\frac{1}{4}$	*m*	$\frac{3}{4}$	*b*
1	Control 20 cm 4/VI	46	73— 65	100	135	105	80	60	135	175	175	130	115
2	Napht. $\frac{1}{2}$ 20 cm 23/VI	170	41— 52	80	170	165	170	220	95	225	225	335	295
3	Napht. $\frac{1}{2}$ 20 cm 4/VI	140	81— 96	185	270	220	255	320	195	290	265	360	370
4	Napht. $\frac{1}{2}$ 20 cm 4/VI	124	94—102	330	465	375[1])	320	350	250	455	505	590	485
5	Napht. 1 20 cm 19/VI	139	70— 83	335[1])	580	380	305	290	315	505	460	485	455
6	Control Vict. Basin 30/VII	51	115—158	50	80	100	95	90	75	120	180	215	150
7	Control 200 cm 1/VIII	152	104—175	40	65	110	130	115	70	140	150	190	185
8	Napth. $\frac{1}{4}$ 150 cm 2/VII	310	158—230	100	115	195	385[1])	200	145	165	285	445	305
9	Napht. $\frac{1}{8}$ 150 cm 30/VII	300	115—180	70	160	215	275	290	110	225	315	375	470
10	Control Vict. Basin 19/IX	42	48— 71	65	150	120	110	60	135	305	240	210	145
11	Napht. $\frac{1}{4}$ 40 cm 17/IX	370	50— 74	225	435	520	560	315	175	330	360	435	430
12	Contr.; seedling 5 cm 25/VIII	8	22— 29	85	90	105	90	145	85	105	135	120	140
13	Napht. $\frac{1}{4}$: seedl. 200 cm 16/IX	137	22— 28	105	270	370	365	225	110	275	310	295	340
14	Napht. $\frac{1}{4}$; seedl. 200 cm 16/IX	90	23— 29	135	385	460	460	380	190	370	355	525	350

TABLE 6*b*. Nymphaea Boucheana.

1	Control 20 cm 3/VI	29	25— 25	80	195	175	205	190	105	210	185	210	225
2	Napht. $\frac{1}{2}$ 20 cm 4/VI	140	27— 30	230	365	290	330	230	240	320	330	340	290
3	Napht. $\frac{1}{2}$ 20 cm 28/V	77	41— 48	210	590	465	460	450	240	520	500	585	370

TABLE 6*c*. Nymphaea Devoniensis.

1	Control Vict. Basin 29/VIII-41	42		65	110	90	65	100	80	170	130	100	140
2	Control Ghent 1939	50		27	110	150	135	135	92	110	210	160	195
3	Napht. 3 Ghent 1939	53		134	380	385	350	230	200	315	405	325	370
4	Napht. $\frac{1}{4}$ 40 cm 18/IX-41	213		180	170	220	270	235	175	170	210	245	265

TABLE 6*d*. Nymphaea zanzibariensis rosea.

1	Contr.; seedling 10 cm 29/VIII	19	28— 36	110	190	150	90	80	130	250	210	135	105
2	Napht. $\frac{1}{2}$; seedl. 200 cm 16/IX	226	27— 36	135	270	330	310	200	130	210	320	435	260

TABLE 6*e*. Nymphaea Lotus lilacina.

1	Contr.; seedling 5 cm 29/VIII	10	36— 38	50	55	70	60	62	65	70	95	85	105
2	Contr.; seedling 200 cm 16/IX	60	36— 39	250[2])	395	720	665	365	230[3])	415	565	620	490
3	Napht. $\frac{1}{4}$; seedl. 200 cm 16/IX	199	35— 40	150	430	365	460	230	170	390	390	460	310

a means: above; $\frac{1}{4}$: at $\frac{1}{4}$ from the top; *m*: in the middle; $\frac{3}{4}$: at $\frac{3}{4}$ from the top; *b*: below.
[1]): many newly divided cells; [2]): at $\frac{1}{8}$ from the top 435; [3]): at $\frac{1}{8}$ from the top 315.

before the end of the experiment; its unfavorable condition is neatly revealed by the cell dimensions.

Table 6c, *N. Devoniensis*. In 3 mg/L naphtalene acetic acid growth was equal to that in the control, but the cell dimensions are a warning against the erroneous conclusion that this hormone would have no influence on the longitudinal growth.

Table 6d, *N. zanzibariensis rosea*. The data speak for themselves.

Table 6e, *N. Lotus lilacina*. No. 2 is one of the very rare examples in which cell elongation may be considered as the only means of reaching the definite length of the petiole; indeed it is a specimen which has been in the same unfavorable conditions as No. 14 of *amazonum* and evidently still worse; that in a normally grown petiole, cell division plays its part, is once more neatly demonstrated by No. 3.

These examples can be multiplied; my data are at the disposal of anyone interested.

Two striking facts appear from these measurements. The dimensions of the leafblades (length of middle vein and greatest breadth) indicate that no relation exists between the assimilating surface and the longitudinal growth of the petioles. The cell divisions must take place at a very rapid rate, otherwise the fact cannot be explained that so very few indications of it were visible, esp. in the longest petioles.

VI. *Discussion.*

Has the greatest length, which a petiole of *Nymphaea* can reach, been attained by the several treatments? This question must certainly be answered in the negative. Several species of *Nymphaea* are grown in the basin of *Victoria regia* where they get rich food and high temperature. They show a luxuriant development and notwithstanding the shallowness of the water, ± 20 cm, they form petioles of considerable length. I measured: *N. Lotus lilacina* up to 120 cm, *N. Lotus var.* up to 180 cm, *N. capensis* up to 130 cm, *N. hybrida var. hort. Oberg. Graebner* up to 230 cm. These petioles are much thicker than those grown in my experiments; they take several weeks to reach their full length and therefore, as I expected, their cell dimensions indicate a high degree of division (comp. my observations on *Victoria regia*, Nos 1, 2); the blades are large (middle vein up to 240 mm, greatest breadth up to 460 mm). If we could transport these plants in basins of, let us say, 400 or 500 cm deep, I feel convinced that they could easily outgrow them and more than that. Theoretically we could force the petioles to elongate indefinitely by growing them in shallow basins, but at the same time by preventing the blades from reaching the surface. I have tried this with *Victoria regia* (2); the petioles reached indeed a greater length, but they soon showed signs of decaying and their abnormally long cells were another indication that this treatment is injurious; yet it might be worth while to try it again with the more robust petioles of *Nymphaeaceae*.

Adding alpha-naphtalene acetic acid to the Victoria basin would be another way of testing the growth potentialities and a very tempting one; the quantity of hormone needed would not be enormous, 2 à 2½ gr for a concentration of ⅛ mg/L; it would be interesting to observe at the same time the reactions of other waterplants, of *Victoria* itself in the first place; but just because I was confident of a striking result, the consequences deterred me.

It must be emphasized, however, that my plant material in the controls, though vigorous and healthy, was not grown in such optimal conditions and therefore never reached dimensions exceeding those mentioned in the tables. The tremendous growth, as well the lengths reached as its amazingly rapid pace, were entirely due to the height of the water column and (or) to the hormones.

Of the three substances used, alpha-naphtalene acetic acid has by far the greatest activity in furthering the longitudinal growth of the petioles of Nymphaeaceae. This holds true for quite a number of other phenomena in plant life, e.g. the rooting response of cuttings (VAN DER LEK and KRIJTHE, 12, RAPPAPORT, 14), the parthenocarpy of fruits (HUBERT and MATON, 10, 11; see also 6), the inhibiton of buds in seedlings

(MATON, 13). The sensitiveness of *Nymphaeaceae* is such that I feel intitled to propose a *Nymphaea*-test besides the *Avena*- and *Pisum*-tests. THIMANN and SCHNEIDER (15) give the concentrations in mg/L of alpha-naphtalene acetic acid giving minimal and maximal responses of straight growth of *Pisum*; when we compare these with mine for *Nymphaea* (table 7) we see that the latter is by far more sensitive; and we should not

TABLE 7. Concentrations in mg/L of alpha-naphtalene acetic acid giving minimal and maximal responses of straight growth.

	Minimum	Maximum
Pisum (THIMANN and SCHNEIDER)	0.05	4
Nymphaea	0.016	0.25

forget that the lowest concentration of 0.016 is probably still too high and that, while a maximum response in *Pisum* means an elongation of 30 à 50 %, in *Nymphaea* it may mean one of some hundreds percent. THIMANN and SCHNEIDER rightly point out the dangers of comparing testmethods as the relative activities of the auxins vary widely with the test plants. Notwithstanding this I think that these data indicate all the same that *Nymphaeaceae* may be considered as very sensitive plants worthy of further examination. On the other hand, we should never forget that they present serious practical difficulties: they can only be grown in summer; their size is such that we are restricted to a small number of specimens which are far from being uniform and whose leaves give only partly the right response; growing them under the carefully controlled conditions as are usual with *Pisum* and *Avena* is excluded; seedlings unfortunately appear to be less suitable. Considering all this I am fully aware that my proposal of establishing a *Nymphaea*-test can have hardly any practical value; but all the same circumstances may occur in which future investigators can consider its possibilities.

It is noticeable that the growth of the leafblades is not influenced at all by the hormones; in higher concentrations they show hypo- or epinastic curvature, but in the concentrations optimal for the growth of the petioles, they do not react at all. This is still more striking when we consider the flowerstalks; these do not elongate, neither as response to deepening of the waterlayer, nor to the addition of hormones; they were sometimes somewhat coiled and this was the only indication of their being "aware of" the presence of auxins.

VII. *Summary.*

1. The growth potentialities of the petioles of *Nymphaeaceae* and some other plants are feeble in the early seedling stage; they increase gradually and strongly and decrease only when the plants are getting old.

2. Each leaf shows individually a parallel behaviour and consequently it only gives the utmost responses to external growth-promoting factors when it is at the suitable age.

3. The two main factors inducing a sudden vigorous renewal of growth of the adult petioles are: (1) deepening of the waterlayer; (2) adding of growth substances to the water. (1) is especially efficient when the plants are young; (2) becomes gradually equally important until later on it takes the upper hand and finally can be the only means of attaining the extreme dimensions.

4. At the suitable age both factors combined or the latter alone result in causing amazing achievements of growth, up to 130 and 160 cm in 48 hours, provided the temperature is between 25° and 30°; when it is between 20° and 25° the same result can be reached but it takes more time.

5. Of the three hormones used, heteroauxin has the least influence, beta-indole butyric acid more and by far the most active is alpha-naphtalene acetic acid.

6. The optimal concentration of alpha-naphtalene acetic acid is about 0.25 mg/L

($0.25 \times 10^{-7} = \pm 7 \times 10^{-5}$ mol); in lower concentrations, even in $\frac{1}{32}$ mg/L ($3.1 \times 10^{-8} = \pm 6 \times 10^{-6}$ mol) individual petioles may give responses which are nearly as strong as the average one in 0.25 mg/L. Owing to the imposed scarcity of the plant material it can be ascertained only with some restriction that there exists a relation between the concentrations $\frac{1}{64}$, $\frac{1}{32}$, $\frac{1}{16}$, $\frac{1}{8}$ and $\frac{1}{4}$ mg/L and growth; higher concentrations are becoming gradually more noxious.

7. The sensitivity of the petioles of *Nymphaeaceae* appear to be 10 à 16 times greater than that of *Pisum*; there are, however, practical objections against establishing a *Nymphaea*-test.

8. The longitudinal growth, even the extremely rapid one, is achieved by cell elongation as well by cell division; the first one takes the upper hand whenever a leaf is in unfavorable conditions, but elongation is never the only means of reaching the definite length; this proves especially to be the case in solutions of more than 0.25 mg/L.

9. The leaf blades are not affected by the hormones in low concentrations, nor are, curiously enough, the flower stalks.

I want to express my heartfelt thanks to the Board of Trustees of the University of Leiden, to the Director of the Botanical Institute, to the scientific and technical staffs of the laboratory and gardens, to the Director, the scientific and technical staffs of the State Herbarium, who, with the utmost kindness, enabled me to continue my research work in my present circumstances.

I have great pleasure in thanking the Direction of the N.V. Amsterdamsche Chininefabriek, Amsterdam, who provided me with the growth substances.

Botanical Institute.

Leyden, October 1941.

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Geology. — *On rocks from the isles of Soegi, Tjombol and Tjitlim. Riouw Archipelago. Netherlands East Indies.* By A. VAN WESSEM. (Communicated by Prof. L. RUTTEN.)

(Communicated at the meeting of November 29, 1941.)

Introduction.

The rock-samples, described in this article, are in the collections of the "Mineralogisch-Geologisch Instituut" at Utrecht. They have been collected in 1930 by Dr. P. M. ROGGEVEEN, geologist of the Billiton My. at that time, during a geological investigation in the Riouw-Archipelago and have been donated to the above-mentioned Institute. The Billiton My. kindly put at our disposal the reports of ROGGEVEEN.

The oldest investigation has been done by R. EVERWIJN (1), who visited the isle of Soegi in 1863. He only mentions the occurrence of white and variegated silicified sandstones; his foreman FLEURY detected some iron-containing sand but no trace of cassiterite. Much later, in 1925 A. BOTHÉ (2) publishes a report which contains the results of an investigation for tinstone in the Riouw-Archipelago. By this exploration it has been stated, that on Soegi and Tjombol there exists a slightly folded shale- and sandstone-series. From the S. coast of Tjitlim he reports eruptive-rocks: granitites and a dike of dark porphyritic rock. On Soegi a sandstone was found from which tinstone-grains could be separated. In 1925 BOTHÉ (3) again treats of the geology of these islands. Besides the already mentioned sandstones there occur slates, phyllitic shales, and phyllite-conglomerates. On Soegi and Tjombol the general strike is North-West, this in accordance with other islands of the Riouw-Archipelago.

The description of ROGGEVEEN is much more discursive. Of the sediments he mentions sandstones, many conglomerates, claystones, and their transitions; they are often red. At one side the sandstones pass into conglomerates, at the other side into claystones. There occur quartzitic, mica-containing and schistose sandstones. In S. Soegi carbonized plant-remains have been found in a conglomerate; here, also silicified wood has been found, belonging to *Protocupressinoxylon malayense* (4). According to ROGGEVEEN the pebble-material in the conglomerates has a polygeneous composition: quartz, quartzitic material, sandstone-fragments, claystone, fragments of siliceous slate etc. are the most important components. He regards all these sediments as Trias. The sediments show lateritization, especially on the low coasts of E. Soegi and W. Tjombol. On the E. coast of Tjitlim ROGGEVEEN has collected remarkable pebbles from a conglomerate, among which also pebbles of volcanic rocks. He suggests that these are related to the "syenite-porphyritic" rocks of E. Tjitlim regarded by BOTHÉ as granitites. Our microscopical research proved the possibility of the identity of the effusive pebbles with the volcanics of E. Tjitlim, which, however, are no syenites but quartzporphyrites and related rocks. Our study proved moreover that this conglomerate also contains pebbles of luxullianite, cranodioriteaplite, chert, and a kaolinquartzrock. This conglomerate, thus, is probably younger than the eruptive rocks in the neighbourhood and certainly younger than the tin-ore-bearing rocks of the region: occurrence of Luxullianite in pebbles. It is uncertain, whether this conglomerate has the same age as the other sediments of the islands, regarded by ROGGEVEEN as Trias. If the conglomerate and the other sediments have the same age, then the complex must very probably be post-triassic, as even the tin-granites are regarded as post-triassic. It remains, however, possible that the conglomerate of S. E. Tjitlim is younger than the rest of the sediments. ROGGEVEEN himself suggests this possibility as he mentions the occurrence of a metamorphic sandstone in the vicinity of the eruptive-rocks of S. E. Tjitlim. This sandstone does not occur in our collection (see, however, later-on, sub "Quartzitic Rocks"). ROGGEVEEN found inclusions in the volcanics of S. E.

Tjitlim, of which he suggests that they are also of eruptive origin. In my samples, however, I did not find a single inclusion. In E. Soegi and W. Tjombol ROGGEVEEN has found rocks, which point to the possible occurrence of an older formation. Near kampong Kang (E. Soegi) he found a talcschist and on Poeloe Pait (also E. Soegi) a siliceous

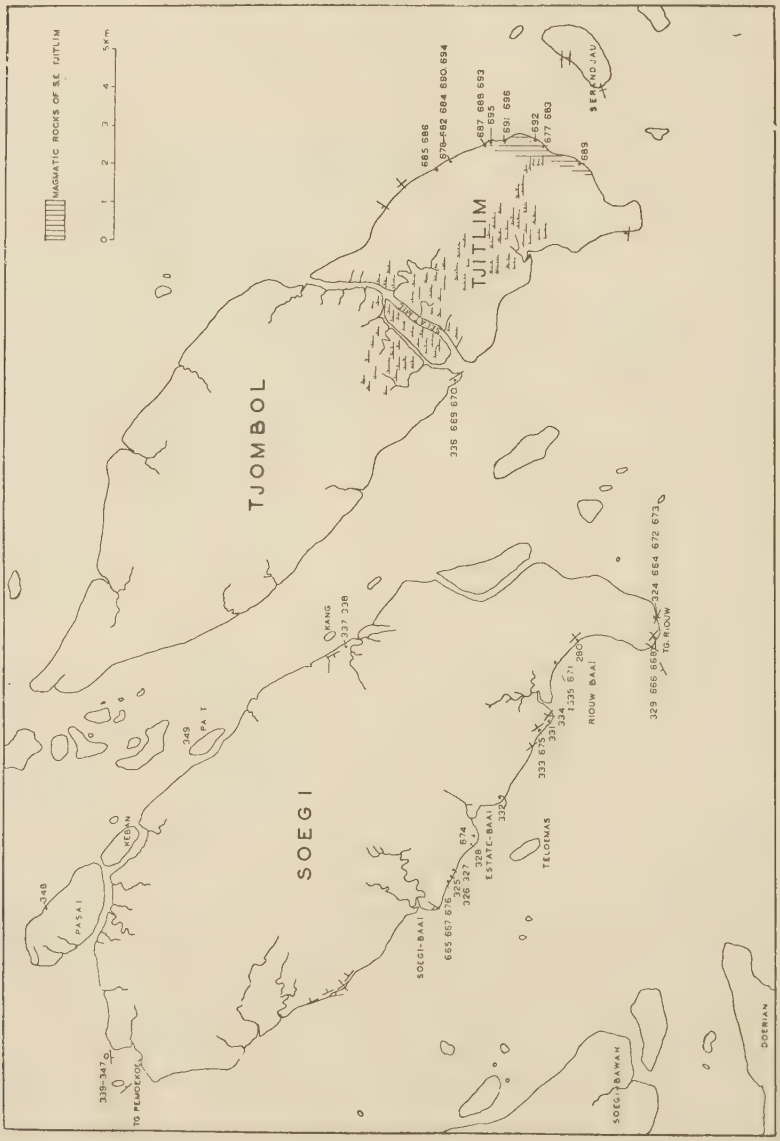

Fig. 1.

talcschist. The microscope proves that they are sericiteschist (Kang) and a quartz-sericiteschist (P. Pait). Moreover he mentions the occurrence of metamorphic sediments and gabbro-diorite from E. Soegi but they have been found in isolated fragments and may have been brought from elsewhere. As stated above, the normal sediments from these islands are regarded by ROGGEVEEN as Trias. The discovery of the *Protocupressinoxylon*

can only prove their mesozoic age. The somewhat metamorphic sediments from E. Soegi are regarded as permocarboniferous; the eruptive rocks of S. E. Tjitlim as jurassic or cretaceous.

According to ROGGEVEEN the tectonics in the region are governed by the Malacca-direction; this is expressed in the topography.

On N. Soegi, however, divergences of this direction occur, here the strike is nearly E.—W. Soegi and probably Tjombol can be regarded as synclines with hard rocks while the strait between them might be an anticline where older, less resistant rocks, have been eroded. Some small faults have been observed in the Soegi-rocks. As compared with the Billiton-sediments, the rocks of Soegi are only slightly disturbed and in consequence their petrographical habit is young. No trace of tin-stone has been found on these islands. Granites do not occur anywhere. The eruptive rocks of E. Tjitlim do not show pneumatolytic influence. The occurrence of granites on Soegi and Tjombol at a small depth is not probable, as the islands lie in a synclinal region. Tinore-veins in the sediments, therefore, cannot be expected, this in accordance with the relative quiet structure.

Description of the rocksamples[1]).

Schists (338, 339, 348, 349).

The samples 338, 339 and 349 which macroscopically are more or less distinctly schistous, show microscopically a directed texture. They are very finegrained. Quartz and sericite are the most important components. In 338 and 339 sericite dominates, in 349 on the contrary quartz is the dominating mineral. Here, sericite appears in sometimes very distinct bands which cause a parallel-texture. In 338 we observe rather many, small, albite-grains, some tourmaline and, on fissures, rather much limonite; it is a quartz- and albite-containing sericitechist. In 339 we find beside quartz and sericite very much epidote; it is a sericite-epidote-schist. In 349 we observe again some albite and limonite. In this sample, accumulations of rather coarse quartz-grains are not rare. It is an albite-containing quartzsericiteschist. Sample 348 gives the impression of a medium-grained quartzite. It is rather strongly sericitized. It contains, however, numerous bipyramides of quartz, clearly idiomorphic, partly with glass-inclusions, partly corroded and possibly also some sericitized felspar-phenocrysts. The rock, therefore, must be a somewhat schistous, silicified and sericitized quartz-porphyrite or quartz-porphyrite-tuff.

Cherts (341, 342, 344—347, 680, 685, 686).

The first six samples are pebbles from a conglomerate at Pemoekoel; 680 is from the conglomerate of S. E. Tjitlim, while 685 and 686 seem to be samples from an outcropping chert near the aforementioned conglomerate. On the map, however, ROGGEVEEN does not indicate at this spot a chert, but conglomeratic sandstone. The samples consist almost wholly of a silicified mass. Locally we observe in 685 and 686 nests and veinlets with coarser quartz or with chalcedony, often in spherulites (341, 685). In 346 there lies a green flush over the silicified mass. This is caused by a pale-green pleochroic mineral; refraction and double-refraction point to chlorite. Sericite we find in small quantities in 345, 680, 685 and 686. It occurs in 680 among the coarser quartzgrains, which have recrystallized in seams in a later stage; the sericite congregates in the axis of the veinlets. That might point to a pneumatolytic influence. Possibly very small grains of a coaly substance are present in 685 and 686. Magnetite and limonite are rare. A certain lamination can be distinguished in 343. Unmistakable radiolaria occur in 341 and 347 and possibly also in 685.

Quartzitic rocks (324, 340, 687, 688).

With regard to the structure only 340, a pebble from a conglomerate, can be called

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[1]) The numbers of the samples correspond with those of the yearcatalogue of 1941 of the "Mineralogisch-Geologisch Instituut" at Utrecht and with those of the map.

a true quartzite. It consists almost wholly of small interlocked quartzgrains, a single muscovitescale, much magnetite and some limonite. 324 is a white-yellow, finegrained rock with many parallel quartz-veins. Under the microscope it presents a fine-grained aggregate of partly spherulitic quartz-crystals with many veins, which are built up by large quartz-crystals. The sample seems to be a transition-rock between chert and quartzite. The two other samples (687, 688) which come from the eruptive zone of S. E. Tjitlim represent possibly the "metamorphosed sandstone" mentioned by ROGGEVEEN. They equally consist of a fine-grained siliceous matrix with quartz-veins; these contain fine crystals of haematite, partly altered into limonite; in the matrix occur many fragments and crystals of pyrite. The origin of these rocks is uncertain; with regard to the mineralization it may be supposed, that they are related to the magmatic rocks in the vicinity. There is, however, no indication that they are metamorphosed sandstones.

Shales (328, 667, 676).

All these samples are from the W. coast of Soegi; we observe in these gray, fine-grained, clastic rocks a more or less distinct stratification, especially in 328. Quartz is always present in angular or in more rounded grains. In 667 we find felspar in small fragments. Sericite in small scales and rare muscovite represent the micas. Chloritous substance occurs in 676. Opaque complexes are caused by kaolin and iron-ore (667). Coal in minute particles, some limonite and very small rutile-needles can be distinguished in 328. Nr 667 contains very much glauconite; it may be called a glauconitic shale.

Sandstones (343, 668).

They are red-weathered rocks. In contrast with 343, 668 shows a distinct stratification. Quartz is by far the most important component; the grains are often typically rounded. Sericite is present in small quantities; in 343 sericite is the result of weathering of rare felspar-grains. Muscovite and felspar are present in only small quantities (343). In 668 opaque complexes are probably caused by kaolin-accumulations and iron-ore. Chlorite occurs in small quantities (668), limonite in small aggregates (343). The occurrence of epidote, tourmaline and zircon in 343 is remarkable; the thin-section presents at least 4 well-rounded grains of tourmaline, partly green, partly blue. The zircons are rather large, max. 150 micron.

Graywackes (325, 326, 327, 330, 332, 333, 334, 665).

Most samples are grey, partly well-stratified rocks; one sample (330) is red-weathered. The samples are characterized by a distinct clastic structure, which changes from coarse-grained (326, 327, 333, 334) to fine-grained (325, 330, 332); quartz is an important component; mostly it is rounded, but in some samples it is angular or even splintery (325, 665). The quartz is sometimes cataclastic and shows almost ever undulatory extinction. Sericite is present in large quantities. In some cases its arising from felspar can be observed very well (330, 333, 334). In 326 and 327 the sericite is arranged in garlands around the quartz. Muscovite-plates are numerous and sometimes showing bent and broken lamellae. In 334 albite-grains can be observed. Orthoclase is always sericitized. A particular place is hold by large fragments of silicified material. These fragments are sometimes rounded and they are very fine-grained: they are chert-fragments. Clastic quartzite-fragments are also present, while in 327 a fragment of porphyritic material can be observed; it is very likely the rock of 348. Further we find biotite (333), rare tourmaline (325, 333), chlorite and epidote (325, 332, 333, 334, 665) and zoisite (334, 665). Ores, e.g. magnetite (325, 326, 327, 332, 333), sulphur-iron-combinations and limonite are not rare and in 330 limonite is very common. The matrix of the rocks contains as recognizable components quartz, sericite and silicified material. The occurrence of numerous chert-fragments and of the porphyritic material in 327 prove, that the graywackes belong to a younger part of the sediment-formation, than the cherts and that they are younger than the schists.

Graywacke-sandstones (329, 336, 666, 669, 670).

All these red-weathered, coarse-grained, clastic rocks are from S. Soegi and S. Tjombol. The most important component is quartz; it occurs in rounded, but also in angular, often undulatory, sometimes cataclastic grains. The larger quartzgrains are cemented either by fine secondary quartz (336) or by small sericite-scales (666, 669, 670). In 666 we observe an intergrowth of quartz and sericite, which makes their arising from felspar probable. Muscovite in small quantities (329); chlorite occurs in 660. Rather coarse grains of tourmaline we meet in 329, 666, 669 and 670. Epidote (329) and opaque complexes like kaolin (666) and iron-ore (limonite) (329, 336, 666, 669), and magnetite (329) occur. Fragments of fine-grained silicified material in 329 and 669 give these rocksamples the appearence of graywackes. The nrs. 669 and 670 are strongly pressed. It is remarkable that the quartzitefragments in 670 have not yet adapted themselves to the prevalent press-direction of this rocksample, which is indicated by the arrangement of the sericiteplates.

Graywackeconglomerates (331, 335, 671, 675).

These gray, coarsely to finely conglomeratic rocks are all from S. W. SOEGI. Large chertfragments, in which in some cases rests of Radiolaria have been conserved, take, especially in 331, an important place. Quartz occurs sometimes in very large rounded grains; they are often cataclastic or show an undulatory extinction. Quartzitefragments with beautifull "mosaic texture" can be observed in 331. In the nrs. 331 and 335 we observe large slatefragments which show a directed texture; their components are small quartzgrains, sericite, some felspar, chlorite, coaly substance and traces of iron-ore. In the cement quartz, sericite and silicified material occur; the cement is locally green-yellow coloured by limonite and chlorite. In 675 conglomeratic bands alternate with bands which wholly consist of quartz; the latter probably are veins. Nr. 675 contains many foreign pebbles. Besides the already mentioned components we stated the presence of: 1) isolated plates of biotite; 2) some grains of what is considered to be staurolite; 3) crystals of plagioclase; 4) many grains of groundmass of porphyritic material (their plagioclases show often fluidal arrangement); 5) grains of corroded quartz, which are to be regarded as phenocrysts of a porphyritic rock and 6) grains of epidote-zoisite.

Porphyrites, Quartzporphyrites and their tuffs (664, 672, 673, 677, 683, 684, 690, 691, 692, 694, 695, 696).

The first three are from S. Soegi; the other ones from S. E. Tjitlim, partly from a conglomerate. All are characterized by a more or less distinct porphyritic texture. Later-on, however, processes have been active which have effaced the porphyritic texture to a certain degree. In 672, 690, 694, 695 and 696 f.i. we observe a strong silicification.

Nrs. 690, 691, 692, 695 and 696 are porphyrites. Among the phenocrysts oligoclase-andesine occupies an important place. Some are fresh, other ones show a profound alteration. In 690 and 696 it is almost wholly sericitized. 695 is wholly silicified, so that the felspar can only be recognized by its outlines. In 691 the felspar is replaced by calcite; in some phenocrysts the beginning of this alteration can be observed. Amphibole- and biotitephenocrysts can only be identified by their outlines. They are often wholly replaced by chlorite, limonite etc. The lavenderblue polarizationcolours of the chlorite in 691 point to pennin. Unimportant are apatite (690, 691), sulphidic ore (692) and titanite (691). In 695 and 696 we observe much magnetite. The fine-grained groundmass is composed of plagioclase, which is more acid than the plagioclase of the phenocrysts, quartz and probably some orthoclase.

The nrs. 690, 695 and 696 are altered porphyrites. They are strongly silicified, sericitized (690) and show some mineralization. In 695 a small vein, filled with heulandite is present, in which, very remarkably, nice idiomorphic magnetite-crystals are enclosed.

The nrs. 664, 677, 683, 684 and 694 are quartzporphyrites; 672 and 673 are quartz-porphyrite-tuffs. The phenocrysts are: quartz (sometimes corroded), plagioclase (more

or less sericitized, probably albite-oligoclase), amphibole and biotite, for a great part replaced by chlorite and limonite. Apatite and iron-ore occur, but only in small quantities; in 677, however, there is much ore. In 677 and 683 the groundmass is holocrystalline; it consists of plagioclase (likely albite), quartz and orthoclase. In 664, 673 and 684 we find a hyaline matrix. In 664 and 673 the groundmass has become partly crystalline, while in 672 and 684 the whole matrix is devitrified. Considering the composition, graindimensions, the presence of splintered quartz etc. of 672 and 673 we call them quartzporphyrite-tuffs. As weatheringproducts of the phenocrysts and the groundmass-components kaolin, limonite, sericite and chlorite may be mentioned.

The three samples from Soegi agree well inter se by the existence of a hyaline matrix and the absence of felspar phenocrysts; they are quartzporphyrites.

The components of the conglomerate from Tjitlim are also quartzporphyrites, which differ from the foregoing three by the presence of large silicified felspar-phenocrysts; the groundmass is here equally silicified.

The quartzporphyrites from the volcanic complex of Tjitlim present less quartz-phenocrysts and more felspar-phenocrysts. Porphyrites without quartz have only been sampled from the volcanic outcrop. Thus, it is possible that the pebbles from the conglomerate have been derived from the volcanic outcrop, but the microscopic difference between the samples is so great, that we cannot regard this relation as proved.

Luxullianites (679, 682).

These samples have been found in the conglomerate of S. E. Tjitlim. They are granular rocks are built up by the following minerals: quartz (often clear and with many liquid-inclusions; in 682 beautiful transverse sections of idiomorphic crystals; in 679 and 682 large quartzgrains with felspar-like circumference and with relics of lamellation), tourmaline (in large quantities, its colour changing from colourless and light-yellow to dark-brown or nearly black; beautiful tourmaline-suns can be observed), apatite, topaz, zircon and possibly some cassiterite, a single muscovite-plate in 679; further some ore, probably magnetite, in 682. As the felspars have been altered into quartz, we must call these rocks silicified Luxullianites.

Diorite (693), found as isolated rock in S. E. Tjitlim.

The sample is holocrystalline, and strongly altered. We can distinguish the following minerals. Plagioclase: only rarely rests of crystals with lamellation can be found; nearly all felspar is sericitized; it is probably albite-oligoclase. Kaolin fills the weathered plagioclases. Muscovite occurs in rather coarse scales. Biotite is the dominating dark mineral. It shows strong pleochroism. Chlorite and complexes of limonite can be regarded as its alteration-products. Amphibole is in minority with respect to biotite. It sometimes still shows distinct traces of idiomorphy; its colour is pale-green, only a little pleochroitic; it has been partly replaced by chlorite. Finally we observe some topaz, also apatite and rather much haematite. Obviously the veinrock 689 belongs to the sequence of this diorite.

Amphibole-dioriteporphyrite (689).

This sample is from S. E. Tjitlim. Its structure is holocrystalline porphyritic. In a groundmass, which shows a microgranitic texture, and which consists of plagioclase, quartz, orthoclase and some amphibole, the following phenocrysts can be destinguished: plagioclase (index of refraction and lamellation point to albite and albite-oligoclase; the felspar is sericitized and epidotized), amphibole (in pale-green idiomorphic crystals), chlorite, epidote and some calcite (can be regarded as alteration-products of amphibole). Besides amphibole, a pale-yellow augite occurs (probably diopside). Haematite in rather large quantities and finally some titanite.

Granodioriteaplite (678).

This is a pebble from the conglomerate of S. E. Tjitlim; its structure is holocrystalline.

The slide consists for the greater part of a typical granophyric intergrowth of quartz and an acid plagioclase. The plagioclase is often dusty and somewhat sericitized. There also occur plagioclases in idiomorphic to hypidiomorphic grains; these show a distinct twinning and are also somewhat dusty; they are oligoclase-andesine. Zoisite, epidote and sericite appear as its alteration-products. Microcline in hypidiomorphic crystals which show gridiron-twinning. Some quartz-crystals are idiomorphic. Biotite occurs in rare green-brown plates, also in radiated aggregates; its pleochroism is not strong and locally it has been altered into chlorite. In the biotite we observe rutile in the shape of sagenite. Finally some zircon and apatite may be mentioned.

Veindeposits (337, 674).

337, clearly a part of a quartzvein, is, according to its location, associated with the schistous rocks of E. Soegi. 674, a quartz-limonite-rock, with very fine idiomorphic crystals of quartz, is equally a part of a vein.

SUMMARY.

The rocks from the islands under consideration belong to different formations.

1. It is fairly probable that the schistose rocks among which a schistose quartzporphyrite are the oldest and that they belong to the permocarboniferous.

2. It is not quite clear whether chert-samples have been found in situ or that they are all pebbles from the conglomerates at Pemoekoel and Tjitlim. It is possible that the cherts 685 and 686 have been taken from outcropping rocks but the fact that ROGGEVEEN indicates at this locality "conglomeratic sandstone" makes the supposition somewhat doubtful. Thus, it is uncertain, whether a chert-formation occurs on the islands.

3. The most important formation of the islands is a sequence of sandstones, quartzites, graywackes, graywacke-sandstones, graywacke-conglomerates and shales. When unweathered they are all gray rocks with only traces of metamorphosis (f.i. the graywacke-sandstone 670, see above). It is probable that the quartzporphyrites from S. Soegi are intercalations in this formation. The following can be said about the age. The absence of general metamorphosis proves that the formation is younger than the permocarboniferous. The presence of pebbles and rounded grains of radiolarite and chert in many of the clastic rocks proves that the formation is younger than the cherts; it is, however, possible that the difference of age is only slight. In some of the rocks grains of tourmaline have been found. As tourmaline is among the heavy minerals of clastic rocks everywhere an ordinary component, we do not feel justified to say that its presence proves that the formation is younger than the tin-ore formation of the region. If, however, the conglomerate of S. E. Tjitlim, with its pebbles of luxullianite and granodiorite-aplite belongs to the graywacke-sandstone formation, it will be justified to regard, at least a part of the clastic formation, as younger than the tin-granites. From the field-observations of ROGGEVEEN it cannot be deduced whether this conglomerate really belongs to the clastic formation. Perhaps Mr. ROGGEVEEN, when returned to Europe, will be in the possibility to give certainty about this important question. The formation is generally regarded as Trias, and, in this case, must be older than the tin-granites. Besides radiolarites and chert there occur most interesting foreign pebbles and grains in the rocks. Material of porphyritic rocks, quartz-phenocrysts of quartzporphyrites (or porphyries), rounded fragments of quartzites, large plagioclases, crystals of biotite and staurolite give indications about the nature of the rocks which have provided the clastic material.

4. In S. E. Tjitlim an outcrop with volcanic rocks has been found. They consist of quartzporphyrites, porphyrites, a diorite (found only as isolated fragment) and an amphibole-diorite-porphyrite. It is very probable that all these rocks belong together. ROGGEVEEN's supposition that they would have metamorphosed sandstones is not proved by our microscopical research. It is possible that the porphyrite-pebbles of the conglomerate of S. E. Tjitlim belong to this sequence; this, however, cannot be proved, as there exists undoubtable differences between the pebbles and the outcropping volcanic rocks.

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Geology. — *On the rocks of the isle of Koendoer, Riouw-Archipelago, Neth. East Indies.* By J. H. GERMERAAD. (Communicated by Prof. L. RUTTEN.)

(Communicated at the meeting of November 29, 1941.)

Introduction.

In 1930, Dr. P. M. ROGGEVEEN, geologist of the Billiton Mij. at that time, carried out a geological investigation in the Riouw-Archipelago. The rock-samples, collected by him, have been donated to the Mineralogisch-Geologisch Instituut at Utrecht whereas the Billiton Mij. kindly put at our disposal the reports of ROGGEVEEN. The rock-samples from Koendoer have been examinated by the author.

In 1863 EVERWIJN and FLEURIJ (1) made a flying survey in the island. According to EVERWIJN, Koendoer is situated on the NW or W of Karimon and has a smaller size (sic.). He already knows the occurrence of granite and sandstone, also of tourmaline and some tinstone. From the mouth of the Tameras he reports magnetite, but probably this has been wolframite.

By exploration of BOTHÉ, BOERS, KROES and LOTH (2) it has been stated, that contact-slates exist at Bt. Penindjau (Bt. Salak) N of the granite. In the Tameras and the Timon-valley tinore has been deposited in placers; in the "kollong" of the latter a quartzvein with tinstone (S 105, D 80 N) has altered the granite into greisen. Possibly EVERWIJN has taken this greisen for a sandstone. Parallel to this quartzvein many tourmaline-strings with tinstone occur in the granite; in the Sg. Limau and the placers of Sg. Gemoeroe and Sg. Koendoer tinore has been found. According to their estimation the quantity of tinore is considerable.

According to the detailed report of BOTHÉ in 1925 (3) Koendoer is very flat; in the N low hills exist and slowly streaming brooklets empty into the sea with wide mouths; the watersheds have nearly disappeared. S Koendoer is one large swamp, but there are small hills of granite in the southernmost part. BOTHÉ mentions the occurrence of Trias. He knows granite, granitite, tourmaline-granitite, pegmatite and amphibole-granite; tourmaline, topaz and tinstone indicate pneumatolysis. According to GISOLF the rock-samples belong to the upper part of the batholith. There are also quartzmica-rocks, considered as differentiations of the magma. A quartz-greisen in the Timonvalley is a pneumatolytic product of the granite; it contains tinstone, tourmaline, mica and quartz. The strike of the above mentioned quartzvein is now given as 150°, probably a misprint. Wolframite occurs abundantly.

The exploration of ROGGEVEEN was much more extensive. The eldest rocks are the crystalline schists of Songkop, perhaps Prae-Carboniferous, because the Carboniferous of Malacca is less regional metamorphic. They are finegrained hornblende-schists with constant composition: hornblende, quartz, epidote and in smaller quantity sphene and sulphidic ore. As a consequence of the directed position of the hornblende crystals the rocks are schistose; the quartz appears in thin, lenticular layers. The rocks show fine, minute folding, visible in thin sections. Possibly these rocks are homologous with those of Merak, Temblas and Tg. Malolo (Karimon). By our examination it has been stated, that they contain only a small quantity of quartz, but much albite; no epidote but zoisite. Squeezed rests of oligoclase-veins also occur (214). According to ROGGEVEEN these amphibolites are paraschists, but we are of the opinion that they are orthoschists. On Koendoer the Carboniferous is absent.

Probably, the rocks regarded as Trias cover the Prae-Carboniferous discordantly. The absence of regional metamorphosis in the Trias proves the higher age of the amphibolites. Its contact-metamorphosis proves that it is older than the granite. The absence of granite-

material in the Trias-conglomerates on Toelang indicates the same. The triassic rocks of this isle are red conglomerates with much quartz (elements up to 5 cm), with lenticules of schisty quartz-sericite-shales and sandstone. On Seraja clay sediments exist. In trenches on the Bt. Djerat weathered, contact-metamorphic clay-sediments occur. According to

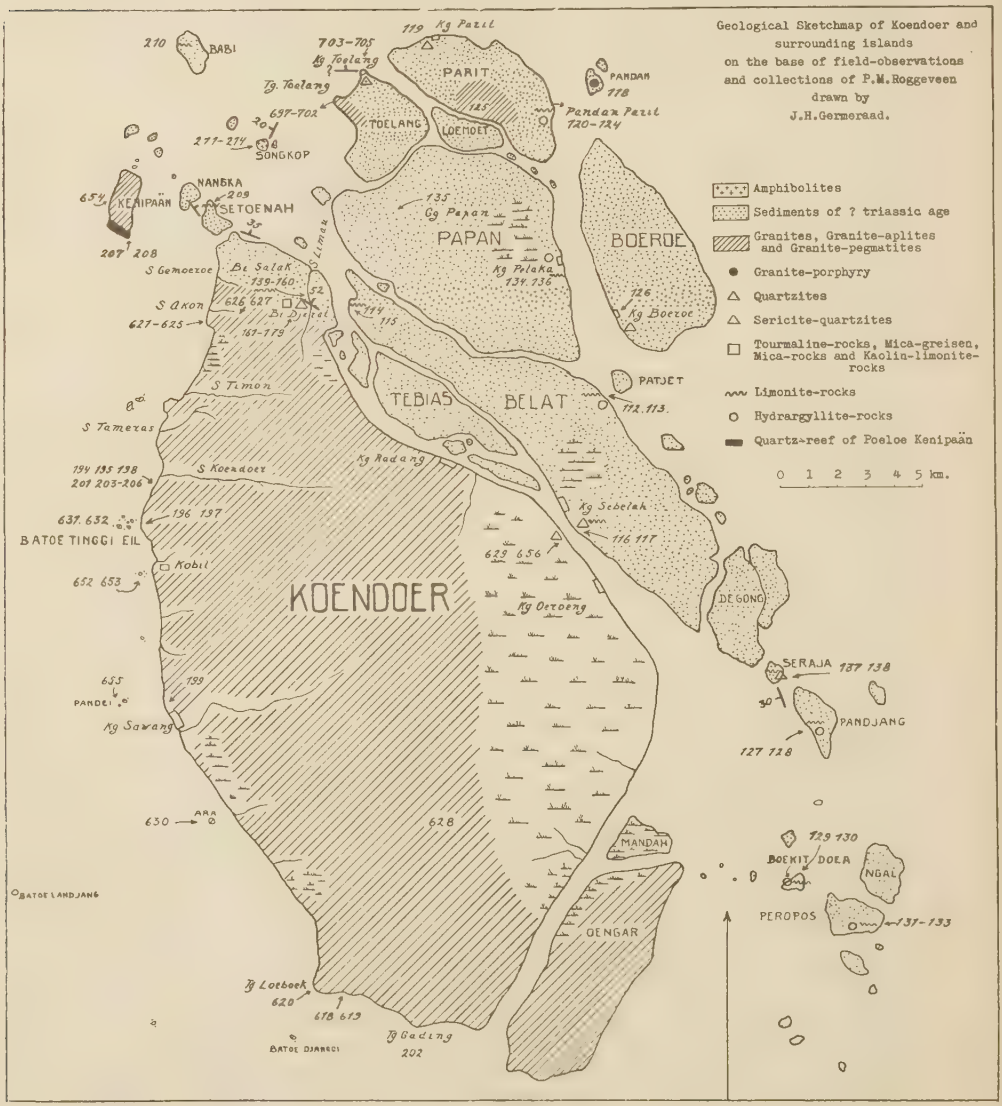

Fig. 1.

The indication △ for sericite-quartzites in the explanation of the map should be ⚠.

ROGGEVEEN they are chiastolite-schists and knotted schists, but they do not appear in the examinated material. The other triassic sediments in this region are intensely lateritic, e.g. on Belat, Pandjang, N. Koendoer, etc. The granite-samples of this region never show this alteration. Sometimes limonitebanks indicate the bedding of the original sediments, e.g. in the neighbourhood of Setoenah. On the beach of Parit at Kg. Parit many rounded

quartzgrains occur, probably a relic of the N Toelang-formation. On the N slope of the Gg. Papan some weathered pieces of schists and veinquartz were observed. At Kg. Boeroe on Boeroe finegrained, silicified rocks exist. The age of all these rocks is not certain. The resemblance with the fossil-bearing rocks on Malacca, Singapore and Lingga makes a triassic age probable. At any case they are older than the granites.

Many granites and connected rocks have been sampled by ROGGEVEEN; their description follows later on.

It is pointed out by ROGGEVEEN, that the tinore has been mainly observed in the granites and for a small part in the contact-zone. In normal sediments no tinore has been found; possibly this is caused by the relatively slight folding, making the sediments impermeable for the tin-bearing fluids. Quite different are the vertically folded sediments on Billiton.

ROGGEVEEN observes that many phenomena prove the tectonic analogy between Malacca and Riouw. They are: 1^0. direction of the sediments is about NW—SE; 2^0. strike of the quartzdike of S Kenipaän ca 130°; 3^0. topographic directions often NW—SE, e.g. on Belat; 4^0. strike of the contactplane of granite and sediments in N Koendoer ca 115°; 5^0. many greisen strings have the same directions; 6^0. slightly parallel position of the orthoclase-crystals in the granite itself with direction about 115—160°.

The batholith of Koendoer can not be connected with the granite-massif of Karimon, which is proved by the different facies of the rocks. The rocks and isles of granite between Koendoer and Sumatra indicate the probable continuation of the Koendoermassif below Sumatra. On the E-coast of Sumatra the granite-hill Senggoenoeng proves this suggestion. Possibly the quartzporphyry (after our opinion: granite-porphyry) of Pandan is a roof-intrusion of the Karimongranite.

The very distinctive directions of the crystalline schists of Songkop may be relics of an older folding, as ROGGEVEEN suggests. Remarkable is their direction of the minute-folding, the strike of which coincides with the direction of Malacca.

Description of the rock-samples of Koendoer, collected by ROGGEVEEN [1]).

Amphibolites (211, 212, 213, 214).

Finegrained, schistose, green-gray rocks with thin lightcoloured veins. In 211—213 green amphibole (pleochroism: lightgreen-bluegreen) dominates, while albite and zoisite occur in much smaller quantities. In 212 zoisite is so abundant, that we may speak of zoisite-amphibolite. Quartz is very rare; accessory are small titanite-grains ("ant eggs"). In all these rocks we observe a strong schistosity. The metamorphosis has taken place in the epizone and is typically regional. Sample 214 consists chiefly of oligoclase in large grains, which are slightly sericitized, just as the albite in the amphibole-bearing part of the thin-section. It also contains some chlorite. In connection with the Temblas-, Merak- and Karimon-samples we may conclude, that this is an altered plagioclase-vein, strongly squeezed by dynamic forces. The amphibolite is considered by us as a metamorphosed gabbro, just in contrary with the opinion of ROGGEVEEN, who speaks of para-schists.

Quartzites (117, 119, 126, 138, 142, 143, 148, 149, 629, 656).

These rocks, strongly varying in colour are all characterized by the domination of quartz. The quartzgrains join together without leaving interspace; they extinguish undulatory or they are cataclastic. Some of them are well-stratified (148, 149); they contain rare ore-grains, possibly magnetite. This mineral is abundant in 119, where it is concentrated in zones; a small quantity of limonite is present. Secondary limonite is more abundant in 138, macroscopically a lightbrown rock; epidote, muscovite and calcite are accessorial. Two very dark, purplebrown samples in which the dark layers alternate with

[1]) The numbers of the samples correspond with those of the year-catalogue of 1941 of the Mineralogisch-Geologisch Instituut at Utrecht and with those of the map.

light ones (629, 656) contain much haematite, concentrated in the dark layers. In 656 limonite in concentric radiated aggregates occurs in fissures. Some tourmaline is present in small prisms. In 142 and 143 dark and light coloured layers also alternate; the dark ones consist of big, idiomorphic crystals of pneumatolytic tourmaline (pleochroism: yellowwhite-darkbrown) and some secicite (142), epidote and magnetite (143). Two samples are quartzitic sandstones (117, 126): the originally clastic elements are mostly quartz, for a small part, however, finegrained quartzite. Pneumatolytic minerals are tourmaline and sericite; limonite is secondary.

Tourmaline-rocks (139, 144, 146, 147, 166).

Except 166, they all have a limonitized habit. There is always much idiomorphic and cataclastic (except 166) tourmaline (pleochroism: lightbrown-darkbrown). In the sections the larger prisms are clearly orientated in one direction. In 166 the tourmaline appears in big idiomorphic prisms, together with some non-undulatory quartz. Quartz also occurs (one grain) in 146. Very much limonite was observed in the thin-sections, filling the fissures. In 139 and 146 some epidote-grains occur. Magnetite appears in 146 and 147, haematite in 139 and 147. All the rocks are compact. The origin is always pneumatolytic.

Sericite-quartzites (703, 704, 705).

The samples 703 and 704 are finegrained with some big quartzgrains; in 705 we find a sericitized and an unaltered part. Microscopically we observe strongly cataclastic quartz, originally big round quartzgrains (quartz-sand), which are deformed cataclastically to ovaloids or totally crushed. Streaks of limonite surround the former sandgrains; sericite-strings appear between them and are of pneumatolytic origin, just as the non-cataclastic tourmaline, which has a pleochroism: lightgreen-bluegreen and occurs in small prisms. In 703 and 704 leucoxene has been observed; in 705 much haematite. In this last sample there also occurs a part without sericite, which proves, that the original rock is a pure, strongly cataclastic quartzite. After stress, these rocks have been pneumatolysed.

Micagreisens (162, 163, 164, 167, 169).

Limonitized rocks, mostly schistose, often with tourmaline. Very much muscovite — often bent — is present in big aggregates. Much clear, undulatory quartz often occurs in layers. In all the samples we observe tourmaline, which is abundant in 167 and 179. Of pneumatolytic origin are also polysynthecically twinned ("calcite-like") tinstone crystals, which appear in big grains in 179. All sections show much limonite, bordering on the muscovite. Epidote (162—164, 179) and ore (probably magnetite) are accessory. In 162 we observe a special tourmaline-zone, in 164 a quartzvein. We do not know the genesis of these rocks: weathered, pneumatolytic sediments or pneumatolytic altered orthorocks. All the samples have been collected in the Kollong Sg. Limau.

Mica-rocks (151, 156, 157).

These rocks are closely connected with the micagreisens. Large muscovite in bent crystals is predominant. Besides, much limonite (156, 157) or tourmaline (151), depending from the degree of pneumatolysis. Always some magnetite has been found.

Limonite-rocks (112, 114, 115, 116, 120, 122, 128, 129, 133, 134, 137, 140, 141, 145, 150, 153, 154, 159, 175, 176, 209, 210.

They mainly contain limonite. Besides we observe magnetite? (128, 140, 141, 153, 154, 175, 176), haematite (114, 116, 120, 122, 129, 137, 150, 154, 209), quartz (150, 175, 210; in 145 and 154 a fine grained mass is probably quartz) and sometimes kaolin (112, 120, 128, 210). Hydrargillite, common in fissures of 115, 120, 122, 128, 129, 133, 134, 137, 140, 145, 150, 154, 175) is secondary. Calcite (122, 175) and tourmaline (145, 150, 175), which

is always pleochroitic: lightgray-bluegray may be present. Accessory is an epidote-grain in 175. The limonite has been often deposited in fissures, then having a concretionnary structure. The texture of the samples is often compact (112, 114, 115, 116, 122, 128, 129, 133, 134, 140, 149, 175), sometimes however porous (the other samples). The tourmaline indicates pneumatolytic influence; the mineral is only found in the samples from the contact in the Bt. Djerat and from the Kollong Sg. Limau. The hydrargillite-bearing rocks pass into the hydrargillite-rocks, the tourmaline-bearing ones into the tourmaline-rocks. Closely related to the limonite-rocks is an iron-ore concretion from Parit (123), which is strongly weathered.

Kaolin-limonite-rocks (152, 155).

They are closely related to the limonite-rocks and contain moreover kaolin, which is abundant in 155. Small prisms of tourmaline indicate pneumatolytic influence.

Hydrargillite-rocks (113, 121, 124, 127, 130, 131, 132, 136).

Hydrargillite dominates in these samples; it has partly been deposited in fissures and then shows beautiful twin-lamellation. Besides, a varying quantity of limonite occurs, often in fissures. Tourmaline (in the contact-zone of the granite-aplite of Parit, 121, 124) some epidote (113, 124) and ore (124, 130) are observed. The structure of the samples is mostly porous, sometimes a little compact and often schistose.

Granites (194, 199, 200, 202, 618—621, 625—628, 630—632, 652—655).

Coarse grained rocks, always containing orthoclase, microcline and their perthites with albite or albite-oligoclase. Independant albite crystals also occur, they are always sericitized and sometimes zoisitized (200, 625). Big, clear, mostly undulatory or cataclastic quartzgrains abundantly (in 200 scarcely) occur in the thin sections. In all the samples muscovite and biotite appear, the latter with pleochroism: lightbrown-darkbrown and often with pleochroitic haloes around zircon (not observed in 620, 628, 630, 653) and sagenite-texture of rutile-needles (202, 619, 625, 626). Often the biotite is altered into chlorite. Muscovite is absent in 621, 625, 631, 655; both micas are absent in 200 where the dark components are represented by green hornblende. Pneumatolytic minerals are tourmaline (not in 194, 200, 619, 626, 654, 655), topaz (not in 200, 625, 626, 654), fluorite (194, 618, 619, 632, 652—655) and sericite (not in 200). Accessories are epidote (not in 628, 630, 655), apatite (not in 194, 202, 620, 626, 630, 632, 652, 653), ore (not in 628, 630), garnet (628, 630, 653), leucoxene (627, 630) or titanite (200) and often secondary limonite. In 200 coarse flakes of chlorite occur in the plagioclase. Cataclastic influence is often observed, very strong in 618, in which even the felspars are crushed. A pronounced direction is nowhere observed. The texture is always holocrystalline, allotriomorphic to hypidiomorphic granular.

Granite-aplites. a) From Parit (125) and Toelang (697—702).

Finegrained rocks, light coloured (yellow or gray) with dark spots. They all contain much orthoclase, the samples of Toelang moreover microcline and perthitic intergrowth of these two with plagioclase. In 697 some myrmekite. Much, in 125 few, albite-oligoclase, which is strongly sericitized and in 125 also chloritized. The felspars are broken by cataclasis, the clear quartz is broken and crushed. The biotite has been found in rare small crystals and is altered into chlorite. 700 and 702 contain zircongrains; when they lie in the biotite they are surrounded by pleochroitic haloes. Fluorite appears in 697, 698, 700 and 701, thus in most of the Toelang-samples, but not in that of Parit. Neither topaz nor tourmaline has been found, the pneumatolysis therefore has not been powerful. Accessories are epidote (697—702), some ore (125, 697), and leucoxene (698—700). The texture always is holocrystalline, allotriomorphic-fine-granular. A pronounced direction has not been observed.

b) From Koendoer (196, 197, 622—624).

They are much more fine-grained than the Koendoer-granite; 196 is the only coarse-grained sample. Always much orthoclase and microperthite; in 623 moreover some microcline. Besides we observe albite or albite-oligoclase, clear, undulatory or cataclastic quartz, which was originally interlocked. In 196 quite chloritized biotite appears in thin streaks. Always much tourmaline with pleochroism yellow-brown; besides some garnet (196, 624). Pneumatolytic minerals are also sericite (not in 623) and topaz (196); no fluorite. Accessory are epidote (not in 623), ore (196) and chlorite (196). Texture always holocrystalline, allotriomorphic-granular. No pronounced direction occurs. The samples 622 and 623 are aplite-veins in granite.

Granite-porphyry (118).

Fine-grained, yellow-gray rock with large quartz- and felsparphenocrists. Microscopically we observe many, large, corroded quartzphenocrists; besides, phenocrists of kaolinized orthoclase and sericitized albite are present. Chlorite and haematite also occur in the felspar. Nearly always the felspar is twinned. The groundmass contains the same minerals: quartz, orthoclase, albite, sericite, chlorite and haematite; accessory is epidote. The texture of the groundmass is rather porous, microgranitic and relatively coarsegrained. Texture of the rock holocrystalline-porphyritic.

Granite-pegmatites (195, 198, 201, 203, 204, 206, 630).

Always coarsegrained rocks which contain mainly orthoclase and microperthite, sometimes also much microcline-microperthite. The strongly twinned plagioclase is albite. Many large quartzgrains, mostly undulatory and cataclastic (not in 195 and 204). Always sericite (not in 206), tourmaline (not in 204) and topaz (not in 195 and 203). Garnet in 195 and 630. Accessories are epidote (198, 201, 206) and ore (203, 630). In 195 limonite appears between the grains, in 206 chlorite occurs. Always the texture is holocrystalline, allotriomorphic-very-coarsely-granular, with graphic intergrowth. No typical direction has been observed.

Quartz-veins (pp. with Cassiterite and Wolframite) (135, 158, 160, 161, 165, 168—174, 177, 178, 205, 207, 208).

They are mainly composed of clear, undulatory, sometimes cataclastic, coarsegrained quartz. Besides, one or more ores have been observed. 177 and 178 only consist of tinstone, but are probably closely connected with quartz-veins. Cassiterite also appears in 161, 168, 171; in 161 and 168 the second ore is wolframite. This mineral becomes abundant in 160, 161, 165, 168—170, 172, 174. Always small quantities of limonite have been observed. Tourmaline occurs in 158, 161, 165, 168, 170, 171, 173, 205; pleochroism: yellow-brown or blue-gray. In 161 and 165 a small quantity of muscovite has been found. Quartz is the only component of 135. In 205, the only sample collected in the centre of the granite area, nearly the only component is cataclastic quartz; besides we find some tourmaline. On Kenipaän a large quartzreef (its length is given by ROGGEVEEN: 700 m) exists. It was stated microscopically, that the samples 207 and 208 consist only of coarse-grained, clear, cataclastic quartz; it seems possible, that this reef is closely related with the granite-massif of Koendoer.

Summary

Probably the oldest rocks occur on Songkop; they are amphibolites with ortho-character. On Merak, Temblas and Tg. Malolo (Karimon) closely related rocks have been collected; the description of these samples will be published later-on by my colleagues VAN RAADSHOOVEN and SWART. According to ROGGEVEEN these rocks are pre-carboniferous.

At any case they are older than the sediments (quartzite, quartzitic sandstone, sericite-quartzite, limonite rocks, kaolin-limonite rocks, hydrargyllite rocks) of the Northern and Northeastern part of the area. These appear in a NW—SE directed region. Their distribution has been indicated on the added map. According to ROGGEVEEN their age may be triassic, because of their similarity with triassic, fossil-containing rocks on Malacca, Singapore and Lingga. It is very remarkable, that the triassic rocks have been so strongly lateritized, whereas the granite-samples in our collection do not show any lateritization. One might be temptated to suppose, that the lateritization is not a recent or subrecent process, but that it has been pre-granitic. We do not believe, that this supposition would be right. If the triassic rocks have been lateritized in pre-granitic time, they must have outcropped at that time, because laterization takes place at or directly below the surface. The lateritic sediments then must have been buried under a thick cover of younger rocks, as the granitic magma has crystallized as a true abyssal rock, which must have taken place at a rather great depth in the earth's crust. Afterwards this hypothetic cover must have been removed without leaving any traces. In our opinion this supposition is too hypothetical to be permitted, only for the explanation of the lateritized trias at the side of the non-lateritized granite. We prefer to suppose, that the alteration was much easier in the sediments than in the coarsely crystalline granite. Moreover Mr. ROGGEVEEN of course has taken the best samples he could get from the granite and it is possible that at other localities this granite presents stronger alteration. We may add, that on the neighbouring island of Sumatra strong lateritization of different young sediments occurs frequently.

At any case the granite-massif of Koendoer is younger than the ?triassic sediments: at N Koendoer, Parit and Toelang clear metamorphism has been observed. The granite itself has contained much gas in its fluid stage, because tourmaline and topaz have been found at several points in the granite and at the contacts with the sediments of the Trias, while in the granite itself fluorite frequently occurs. In the sediments the tourmaline is absent in all samples which have been collected at large distance of the contact.

It is of interest that the ?triassic sediments, the abyssal rocks and the veinrocks show strong cataclasis whereas no signs of cataclasis can be observed in most of the tourmaline. This proves that the magma intruded during the period when orogenetic forces were very active but that orogenetic action had come generally to an end when the pneumatolytic tourmaline was formed. Only in some samples from the contact zone in N. Koendoer crushing of the tourmalines has been observed.

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Palaeontology. — *Old Tertiary Mollusca from Curaçao.* By C. O. VAN REGTEREN ALTENA. (Communicated by Prof. L. RUTTEN.)

(Communicated at the meeting of November 29, 1941.)

The Mollusca dealt with in this paper were collected by Dr. P. WAGENAAR HUMMELINCK on September 7th 1936 and belong to the collection of the Geological Institution of the University of Utrecht. They all derive from a marly limestone exposed at the steep N edge of the Seroe Mainchi (= Mainsjie), a hill immediately S of the St.-Joris bay, in the island of Curaçao (textfig. 1). The marly limestone is exposed at some 15 m below the top, which has an altitude of 43 m; it is covered by a thin layer of quaternary limestone, and, contrary to the latter, contains a considerable amount of quartz grains. Most of the casts had been loosened from the rock by desintegration and were found at a lower level on the talus. The present locality has been mentioned by HUMMELINCK (1940, p. 21) as ".....probably also an eocene marl in the S, forming the Seroe Mainsjie", but there are no previous records in literature.

Fig. 1.

I received a great number of fossils, mostly casts and impressions of Gastropods and Lamellibranchs for examination. The preservation of the greater part is rather bad, so that only few of them could be identified with some accuracy, and only very few with such accuracy that they give some hold for determining the age of the bed. The samples contain no Foraminifera; apart from the Mollusca there is only one badly preserved fragment of a coral.

The following Mollusca have been identified:

Gastropoda

Architectonica spec., 1 cast, and 1 fragment of a second specimen
Ampullina ? spec., 1 specimen
Polinices spec., 1 specimen

Lamellibranchia

Arca spec., 1 specimen
Venericardia parinensis OLSSON, 1 specimen
Cardium spec., 3 specimens

Clementia cf. *parinensis* OLSSON, 20 specimens
Tellinidae gen. spec., 1 cast
Raetomya arbolensis falconensis (RUTSCH), 2 specimens.

The presence of a *Venericardia* of the *planicosta* LAM. group and of the genus *Raetomya* at once makes it clear that these fossils are of lower palaeogene age. *Venericardia parinensis* has been found in the Salina, Pale Greda, and Parinas formations in Peru (OLSSON 1928), and in some localities in Trinidad presumably belonging to the upper palaeocene (RUTSCH 1936, pp. 174 seq.). According to RUTSCH (1936, p. 180) the lower palaeogene formations distinguished in Peru by OLSSON can be paralleled with the divisions of the conformable period in E North America and Europe as appears from the following stratigraphical table.

Peru	E North America		Europe		
Saman	EOCENE	Jackson	EOCENE	Bartonien s.l.	Bartonien s. str. (= Wemmelien, Ludien)
Talara					Lédien
Restin		Claiborne		Lutétien	
Parinas				Yprésien	
Pale Greda		Wilcox	PALAEOCENE	Landénien	
Salina					
Negritos		Midway		Montien	

In Peru, where the molluscan fauna of the whole palaeocene and eocene series has been investigated, *Venericardia parinensis* does not occur in older strata than the Salina, nor in younger beds than the Parinas formation. Therefore it is probable that this species really did not live longer than from middle palaeocene to lower eocene. Consequently the old tertiary deposit of Seroe Mainchi must be younger than lower palaeocene and older than middle eocene. A lower eocene age is the most likely, as the specimen of *Venericardia parinensis* agrees with WOODS' form C, which is characteristic of the Parinas formation.

Raetomya arbolensis falconensis has been described from some localities in the provinces of Falcón and Lara in Venezuela, and RUTSCH (in SEITZ & RUTSCH 1930, pp. 600—601) thinks it probable that it derives from eocene beds, as the typical *R. arbolensis arbolensis* (WOODS) occurs in Peru from the Parinas up to the Saman formation, that is in those beds which have been paralleled with the European eocene.

Clementia parinensis has only been recorded from the Parinas and Restin formations of Peru, consequently from lower and middle eocene beds.

We saw that the specimen of *Venericardia parinensis* has probably a lower eocene age, and there is no reason not to assume a lower eocene age for the other Mollusca also, but the material is too scanty for us to be absolutely sure about the age of the Seroe Mainchi deposit.

Up to the present rocks of palaeogene age had been found in Curaçao in one place only. They were described as the Seroe di Cueba series by MOLENGRAAFF (1929, p. 25).

The Seroe di Cueba [1]) is situated at a distance of some 40 km NW of the Seroe Mainchi (textfig. 1). MOLENGRAAFF (l.c.) and RUTTEN & VERMUNT (1932) consider the Seroe di Cueba series to be of upper eocene age because of the Foraminifera and Echinoidea it contains and because of the occurrence of *Tubulostium leptostoma clymenoides* (GUPPY) (cf. RUTSCH 1939, p. 240). Casts of Mollusca were collected by MOLENGRAAFF in the basal limestone of this series, but they were too bad for identification. I sought in vain for these casts in the MOLENGRAAFF collection, which is kept in the Geological Institution at Delft. The curator, Dr. P. KRUIZINGA, kindly informed me that they have probably been thrown away, because of their bad state of preservation. It is, however, improbable that any relation exists between these two palaeogene molluscan faunas of Curaçao, as the Seroe di Cueba series must be much younger than the Seroe Mainchi beds.

The following remarks on some of the examined specimens may be added here.

Venericardia parinensis OLSSON

Plate, figures 1*a*—*d*

1928 *Venericardia planicosta* var. *parinensis*, n. var. — OLSSON, Bull. Amer. Paleont., **14**, n. 52, p. 26, pl. 5 fig. 3.

1939 *Venericardia* (*Venericor*) *parinensis* OLSSON. — GARDNER & BOWLES, U.S. Geol. Survey Prof. Paper 189F, p. 183, pl. 40 fig. 3 (with full synonymy).

There is one damaged specimen of this species. Its dimensions are (after careful reconstruction): height 86 mm, length 91 mm, diameter 60 mm. In the middle of the shell the ribs fade always at a distance of about $^1/_3$ of the height of the shell from the umbo; in the anterior and posterior parts they can be followed up farther in the direction of the ventral margin. They number 17 in front of the umbonal slope and probably 24 in total. The right and left valves could not be separated from each other, therefore I sawed the fossil into two halves, exactly in the medial plane, in order to be able to examine the features of the hinge. This appeared to be very heavy. The medial cardinal of the right valve agrees rather well with the diagram given for this species by GARDNER & BOWLES (1939, p. 132), although this must have been taken from a larger specimen. The specimen agrees with WOODS' form C (WOODS 1922, pl. 3 fig. 3) from the Parinas formation, in which the umbos are prominent and much curved, and the sculpture is rapidly fading away, and not with his form B (Ibidem, fig. 2) occurring in older beds.

Clementia cf. **parinensis** OLSSON

Plate, figures 2*a*—7*b*

1928 *Clementia* (*Clementia*) *parinensis* n. sp. — OLSSON, Bull. Amer. Paleont., **14**, n. 52, p. 35, pl. 7 figs. 3, 4.

There are 20 specimens of what is evidently one and the same species of *Clementia*; they are casts sometimes partly covered by fragments of the thin shell. As to the dimensions and habitus they much resemble *Cl. parinensis* from the Parinas and Restin formations of Peru, but the concentric sculpture does not fade away so conspicuously towards the ventral margin as in that species, and the posterior extremity is slightly less truncate. These characters are, however, variable in my material; there are specimens which differ but slightly from *Cl. parinensis* as figured by OLSSON. The dimensions of

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[1]) Mr. HUMMELINCK pointed out to me that this hill should properly speaking bear the name of Seroe Bartool, as the real Seroe di Cueba is a hill situated more to the N at the coast. It is to be regretted that this mistake is made as well on topographical maps as in the geological literature (cf. West-Indische Gids, **21**, p. 89, 1939).

some of my specimens can be compared with those given by OLSSON with the aid of the following table.

	Length	Height	Diameter
Clementia cf. *parinensis*, Curaçao	28 mm	24 mm	13 mm
	27 mm	21 mm	13.5 mm
	26 mm	21 mm	13 mm
	23 mm	21 mm	13.5 mm
Clementia parinensis, Peru	27.5 mm	22.75 mm	14.5 mm

Raetomya arbolensis falconensis (RUTSCH)

1930 *Mya* (*Raetomya*) *schweinfurthi falconensis* nov. var. — RUTSCH, Ecl. Geol. Helv., **23**, p. 597, pl. 16 figs. 2—5.

As the two specimens found by Mr. HUMMELINCK are damaged, their dimensions had to be ascertained after careful reconstruction. The Geological Institution of the University of Amsterdam possesses two more specimens labelled "Curaçao"; it is possible that these shells derive from the same locality. They are nearly entire, and their dimensions could consequently be ascertained without doubt.

Though I have not been able to examine specimens of *Raetomya* from other localities, I got the impression from the study of the literature, that the African forms designed to this genus differ more from the South American subspecies described by WOODS and RUTSCH than the latter differ from each other (cf. SEITZ & RUTSCH 1930 and the literature cited there). Therefore it seems rational to consider the two South American forms specifically distinct from the African ones.

The dimensions of my specimens and those of the subspecies known from Peru and Venezuela respectively have been compiled in the following table. It appears that my specimens agree very well with the Venezuelan subspecies, especially as to the relation between diameter and length.

	Length	Height	Diameter	$\frac{100\ H}{L}$	$\frac{100\ D}{L}$
R. arbolensis falconensis					
1) Seroe Mainchi, Curaçao	51 mm	34 mm	27 mm	61	53
(Geol. Inst. Utrecht)	37 mm	27 mm	21 mm	73	57
2) Curaçao	41 mm	29 mm	25 mm	71	61
(Geol. Inst., Amsterdam)	49 mm	33 mm	27 mm	67	55
3) Venezuela (SEITZ & RUTSCH	49 mm	36 mm	26 mm	73	53
1930, p. 597)	57 mm	42 mm	29 mm	74	51
R. arbolensis arbolensis					
4) Peru (SEITZ & RUTSCH	62 mm	36 mm	19 mm	58	31
1930, p. 597)	59 mm	36 mm	22 mm	51	37

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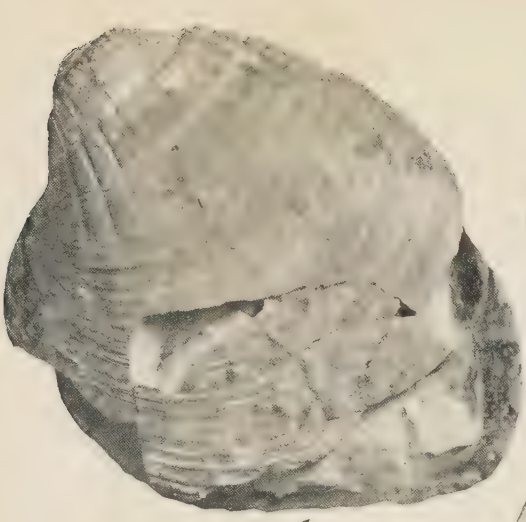
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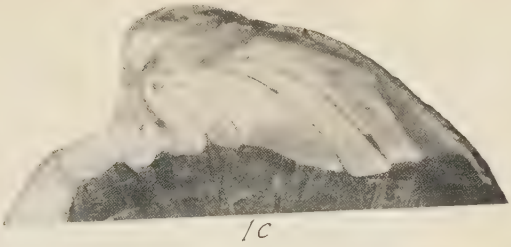
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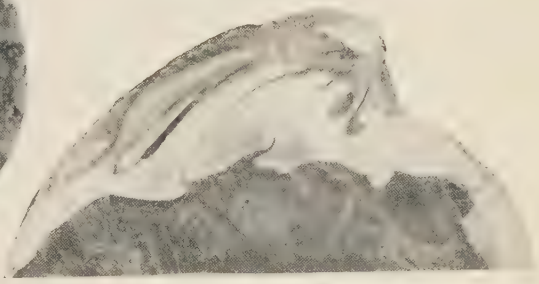
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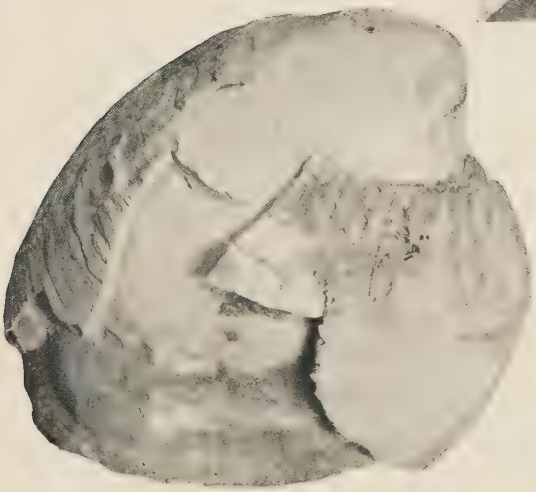
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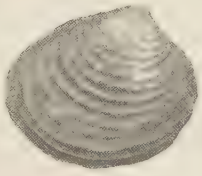
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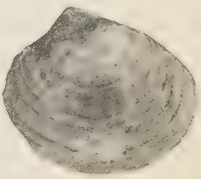
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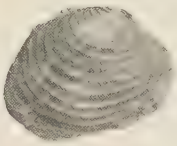
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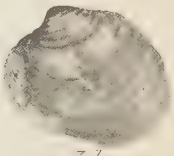
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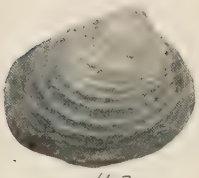
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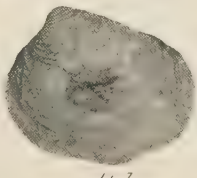
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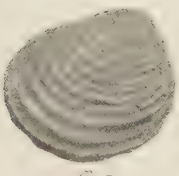
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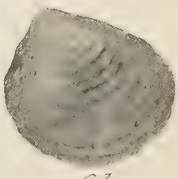
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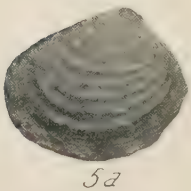
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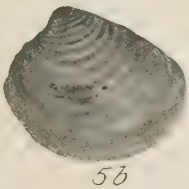
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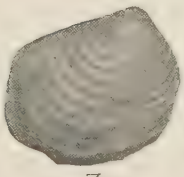
7a

7b

Figures 1a—*d*. *Venericardia parinensis* OLSSON, Seroe Mainchi, Curaçao. *a* left valve, *b* right valve, *c* and *d* medial section through the hinge (diminished, dimensions see text). Figures 2a—7b. *Clementia* cf. *parinensis* OLSSON, Seroe Mainchi, Curaçao. *a* right valves, *b* left valves (natural size).

Comparative Physiology. — *Ueber den Tonus des Schneckenfusses (Helix pomatia L.).* II. *Die Tonuslösung.* (Vorläufige Mitteilung).[1]). Von N. POSTMA. (Aus dem Institut für vergleichende Physiologie der Universität zu Utrecht.) (Communicated by Prof. H. J. JORDAN.)

(Communicated at the meeting of November 29, 1941.)

A. Einleitung.

In der ersten Mitteilung (15) haben wir die, den Tonus erhöhende Funktion der Pedalganglien eingehender kennen gelernt: sie äussert sich in bestimmten Gesetzmässigkeiten bezüglich des **Niveau der Pedalkonstanz.** Neu war das Studium des Einflusses von *Belastungsgrösse* und von *Hydratation* und *Dehydratation* hierauf. Letztere hat eine Steigerung des Niveau zur Folge; reichliche Wasseraufnahme ändert das Niveau dahingegen nicht. Niedrigere Belastung fördert die Konstanzerzeugung; höhere Belastung erniedrigt das Konstanzniveau der Wiederholungskurven.

Wärme, und zwar erst oberhalb 22° C erniedrigt das Niveau; Kälte ändert es nicht. Durch schwache Kokainisierung der Pedalganglien wird das Konstanzniveau herabgedrückt; Kochsalzlösung auf die Ganglien aufgetragen, steigert das Niveau. Diese Versuchsbedingungen wurden nunmehr, im Gegensatz zu früheren Versuchen, vor Applikation der ersten Belastung verwirklicht. Die so erhaltenen Resultate bestätigen im Grossen und Ganzen die Befunde von JORDAN und Mitarbeitern (2, 3, 4, 5, 6, 7, 8) über den Einfluss von Mehrbelastung, Wärme, schwache Kokainisierung und Aktivierung der Ganglien durch Kochsalz.

Die Pedalganglien greifen aber nicht nur durch Steigerung des Tonus ein, die Regulierung erfordert auch eine Abänderung des peripheren Tonus in entgegengesetztem Sinne: also **Verminderung des Tonus.** Diese ist schon ohne Belastung des Fusses zu beobachten (vgl. 1 Abb. 2.a). Von zwei Vergleichspräparaten liegt das noch mit Pedalganglien versehene flach und breit der Wachsplatte auf: d.h. der pedaltonische Fuss breitet sich durch das eigene Gewicht etwas auf der Unterlage aus. Er zeigt geringeren Dehnungswiderstand, hat einen niedrigeren *Tonusspiegel,* bei ähnlicher Länge, d.h. gleichem *Tonusniveau* (vgl. 15 S. 1154) als das ganglienlose Objekt, das hoch und schmal geworden is. DE MAREES VAN SWINDEREN (16) hat gefunden, dass diese Tonuszunahme nach Exstirpation der Pedalganglien sich in etwa 15 min vollzieht. Diese Ganglien erniedrigen mithin normalerweise, und zwar ohne irgendwelchen Reiz den Tonusspiegel, welchen die Peripherie an sich zeigen würde: *die primäre Funktion dieser Ganglien ist Tonuslösung,* wie JORDAN auch auf Grund seiner Erfahrungen an Aplysia (3) dies stets betont hat (4, S. 223).

Werden solche Fusspaare durch gleich grosse Last gedehnt, so zeigt auch das Mechanogramm des pedaltonischen Objektes, auf jeden Fall im Anfang, eine grössere Dehnungsgeschwindigkeit als das Myogramm des anderen Fusses. Bei Anwendung der auxotonischen Dehnungsmethodik behauptet sich der Vorsprung, vorausgesetzt dass man von geringer Belastung ausgeht. War die Anfangslast relativ hoch, so folgt nach einer steilen Dehnungsstrecke der s.g. Umschlag: der Dehnungswiderstand wächst schnell an und die Dehnungskurven beider Präparate schneiden einander, weil der pedaltonische Fuss auf höherem Niveau als der Fuss ohne Ganglien die Konstanz erreicht (JORDAN, 4, S. 226).

Bei Anwendung der isotonischen Dehnungsmethode tritt der „Umschlag" immer ein

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[1]) Mit Unterstützung der Niederl. Regierung aus den Einkünften der „Zomerpostzegels".

und man erhält in 60 % der Versuche eine Ueberkreuzung der Kurven (15 Abb. 1). Der von JORDAN bei der auxotonischen Dehnung gefundene Einfluss der Belastungsgrösse (Niveaux der Muskel- und Pedalkonstanz gegenüber dem Konstanzniveau beim Fuss ohne Ganglien) ist wichtiger als sich damals erwarten liess. Das hat sich nicht nur bei den Versuchen über das Niveau der Pedalkonstanz gezeigt, sondern auch in ausgesprochener Weise bei der Untersuchung der tonuslösenden Funktion.

JORDAN führte bei seinen Versuchen die Abänderung der Versuchsbedingungen erst nach Eintritt der Pedalkonstanz aus, wenn also die erste steile Dehnungsstrecke vor dem Umschlag vorüber war; eine Beeinflussung dieser Strecke wurde daher nicht untersucht. Durch höhere Last, Wärme und Kokain wurde während Konstanz aufs neue Dehnung ausgelöst; Kochsalz hatte Verkürzung zufolge. *Ich habe nun den Einfluss der betreffenden Eingriffe auf die* **erste Dehnungsstrecke** *verfolgt, indem ich die Versuchsbedingungen schon vor Applikation der ersten Belastung einwirken liess.*

Man kann derart feststellen, ob die Tonuslösung sich bei der Belastung des Fusses zu Beginn in einer längeren und sehr steilen Dehnungsstrecke äussert, welche mit einem scharfen Umschlag in Konstanz übergeht, oder ob der Dehnungswiderstand sofort ziemlich hoch ist und die Konstanz relativ allmählich eintritt (Abb. 3 Kurve III). Die so erhaltenen Ergebnisse muss man statistisch bearbeiten: Die Reaktionstypen bis zum Umschlag werden in drei Klassen eingeteilt, je nachdem die sehr steile Strecke deutlich oder fraglich ist, oder aber gänzlich fehlt. Ohne besondere Behandlung der Tiere einer Versuchsreihe von 33 Objekten wurde die Zugehörigkeit zu den drei Klassen zu etwa 61, 18 und 21 % festgestellt. Das entsprach offenbar für dieses Material der normalen Wahrscheinlichkeit. Ich habe daraufhin verfolgt, wie diese Perzentage sich modifiziert wenn die Versuchsbedingungen abgeändert werden, sei es während der Dehnung (Temperatur, Aktivitätszustand der Ganglien) oder nach entsprechender Vorbehandlung der Versuchstiere (z.B. ob sie trocken aufbewahrt worden waren oder ob sie reichlich Wasser hatten aufnehmen können).

Zweitens habe ich den Tonus durch Reizung zerstört. Es war nämlich möglich, durch mechanische Reizung von Nerven oder durch mechanische, oder vielleicht auch chemische Reizung der Pedalganglien die Konstanz zu durchbrechen. Oefters wurde der Durchbruch durch eine kleinere oder grössere Verkürzung eingeleitet.

B. Ergebnisse.

I. *Ueber reflektorische Lösung der Pedalkonstanz.*

1. *Einfluss der Belastung.*

Der beim Fuss mit Pedalganglien schon im Moment der Belastung bestehende niedrigere Tonusspiegel hat eine typisch steile Dehnungsstrecke zur Folge. Dieser Einfluss der primären tonuslösenden Funktion der Ganglien wird nun weiterhin durch relativ hohe Lasten verstärkt:

Versuchsserien *PK–R–L* · *A*, : *B*, · *C* und *–L/G*: Vor Auftritt der Muskelkonstanz (= Pedalkonstanz bei niederer Last, hier 5 g) fand ich nur in etwa 8 % ($^1/_{13}$) der Fälle Tonuslösung, während bei Zentralkonstanz (höhere Belastung, hier 20 g) eine Häufigkeit der sehr steilen Strecke von 77 % ($^{26}/_{34}$) gefunden wurde; eine Belastung mit 10 g nahm eine Zwischenstellung ein: 7 von 12 Objekten zeigten sehr ausgesprochene Tonuslösung. Dass es sich nicht um einen Unterschied im Verhältnis zwischen äusserer Reibung und Last handelt, beweist die zunehmende Abänderung des Niveau der Muskel- und der Zentralkonstanz bei, nach tonischer Verkürzung, wiederholter Dehnung (vgl. 14 u. 15). Niedere Belastung steigert das Niveau in den aufeinanderfolgenden Dehnungsversuchen, bei grösserer Last wird es erniedrigt. Die Reibung in der Dehnungs- und Registrierapparatur bleibt jedoch während der aufeinander folgenden Wiederholungskurven konstant, während bei niederer Last die Niveauerhöhung, bei höherer Last die Niveauverminderung von Kurve zu Kurve zunimmt, sodass diese progressiven *Aenderungen des*

Niveau als Verschiedenheit der Reaktion des pedaltonischen Präparates auf diese Lasten zu deuten sind.

In 8 von 13 Versuchen über Muskelkonstanz zeigten die Myogramme, verglichen mit Dehnungskurven von ganglienlosen Füssen (Netzkonstanz) in der anfänglichen steilen Strecke nur einen, durch den niederen Ruhetonus verursachten Vorsprung: der pedaltonische Fuss lässt sich *passiv* schneller dehnen als der ganglienlose, weil er vor der Belastung einen niederen Tonusspiegel hatte. Bei relativ hohen Lasten kommt dazu noch eine, durch die Dehnung ausgelöste Tonusverminderung, welche die typische sehr steile und längere Anfangsstrecke zur Folge hat. Diese Aufhebung von Tonus durch die Ganglien bekommt man gleichfalls sehr deutlich zu sehen, wenn während Muskel- oder „Mittelkonstanz" die Last hoch gemacht wird (Abb. 1, Kurve 2). Eine ähnliche Reaktion zeigte der Helixfuss im JORDANschen „Halbtierversuch" (2, 3; 5, S. 591; 15).

In Bezug auf die *Tonuslösung* während der ersten Dehnungsstrecke beträgt sich der Fuss also *auch bei der ersten Dehnungskurve verschieden, je nachdem seine Dehnung mit Zentral- oder Muskelkonstanz endet.*

2. *Einfluss von Kälte und Wärme, von Hydratation und vom Aktivitätszustand der Pedalganglien auf die erste Strecke der Dehnungskurve.*

In der Einleitung haben wir beschrieben, wie die, in der Ueberschrift dieses Abschnittes genannten Faktoren das Niveau der Pedalkonstanz beeinflussen. Das Resultat einer statistische Bearbeitung der Ergebnisse in Bezug auf die steile Strecke der entsprechenden Versuchsserien sind in der folgenden Tabelle zusammengefasst:

	Steile Strecke		
	Deutlich	Fraglich	Fehlend
Normal (15°)	61 %	18 %	21 %
Temperatur 2° C.	*20*	20	60
7	*40*	20	40
12	60	20	20
bis 27	60	20	20
32	100	*0*	*0*
Dehydratation	*29*	*29*	42
Sättigung mit Wasser	56	33	11
Kokain	*0*	43	57
Kochsalzlösung	50	50	*0*
Kochsalzkristall	87	13	*0*

Es zeigt sich also, dass, im Gegensatz zur einseitigen Wirkung durch Wärme auf das Niveau der Pedalkonstanz, die Tonuslösung auch durch Kälte beeinflusst wird, unterhalb 12° C wird mit zunehmender Kühlung die steile Strecke stets mehr unterdrückt; durch Wärme wird sie gefördert, wenn auch erst oberhalb 27° C.

Ebenso wie in Bezug auf das Niveau der Pedalkonstanz, hat nur Dehydratation Einfluss auf die Tonuslösung: die steile Strecke tritt weniger häufig auf; Sättigung mit Wasser ändert dahingegen die normale relative Häufigkeit des Auftretens steiler Anfangsstrecken nicht überzeugend.

Schwache Kokainisierung dämpft die tonuslösende Funktion der Pedalganglien: eine deutliche steile Strecke fehlt, sie wird von einer Häufigkeit von 61 % auf 0 % herabgedrückt. Kochsalz, als Kristall appliziert, fördert die Tonuslösung stärker als Kochsalzlösung (15 %). Diesen Unterschied werden wir bei der zusammenfassenden Diskussion eingehender besprechen.

II. *Ueber Lösung der Pedalkonstanz durch Reizung.*

Bei den nunmehr zu beschreibenden Versuchen machten wir Gebrauch von der durch uns (POSTMA, 14; MAAS, 9) angegebenen Technik der Wiederholungskurven. Es gelingt dann jeweils nach ganz widerstandsloser Wiederverkürzung recht gut reproduzierbare Kurven zu erhalten, d.h. die Wiederholungskurve zeigt dieselbe Gestalt und Gesetzmässigkeiten, wie die ursprüngliche, erste Dehnungskurve.

1. *Indirekte Reizung der Pedalganglien durch den N. pallialis.*

Bei den Versuchen über die Pedalkonstanz waren bisweilen nach Abschneidung des Eingeweidesackes nicht nur die Ganglien und die Nn. pedales geschont worden, sondern gleichfalls die Nn. palliales. Wurde nun während Pedalkonstanz einer der Nn. palliales mit kurzem Schlag einer scharfen Schere durchschnitten, so gelang es öfters aufs Neue eine schnelle Dehnung auszulösen (Abb. 1, Wiederholungskurve 3, bei R.). In diesen

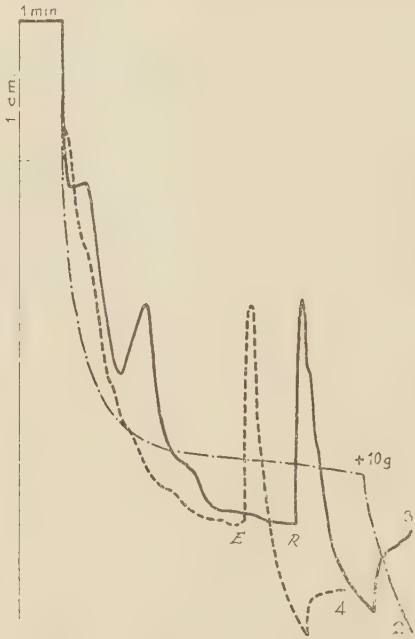

Abb. 1. Wiederholungskurven von einem Schneckenfusse mit intakten Pedalganglien. Objekt *PK–W–MR. F* VI, 21-II-'34. Gewicht des Tieres 18 g, Last 20 g. Ordinaten: effektive Längenänderung des Fusses (Ordinatenschreiber nach N. POSTMA, 1933); Abszissen: die Zeit. Kurve 2: Steile Strecke und Pedalkonstanz nach kurzem „Umschlag"; Erhöhung der Belastung auf 30 g hat Tonusfall zur Folge; 3: Während der Pedalkonstanz wird (bei *R*) einer der Nn. palliales mit kurzem Scherenschlag mechanisch gereizt; es folgt Tonusfall nach schneller Kontraktion. 4: Im Zustand der Pedalkonstanz werden (bei *E*) die Ganglien exstirpiert; die Reaktion entspricht vollkommen der in Kurve 3 wiedergegebenen.

Fällen wurde die Pedalkonstanz durch einen solchen mechanischen Reiz auf genau die gleiche Weise gelöst, wie durch Exstirpation der Ganglien (Wiederholungskurve 4, bei E.). Die erstgenannte Art der Reizung (via N. pallialis) zerstörte die Konstanz nicht unwiederruflich, denn in Kurve 4 erzeugt das Präparat aufs neue Konstanz und zwar

gleichfalls bei Spannungslänge. Dem Durchbruch der Konstanz geht eine Kontraktion voran. Diese Kontraktion nach Pallialisreizung entspricht durchaus derjenigen nach Ganglienexstirpation. Solche Verkürzungen können aber in beiden Fällen viel geringer sein als in den abgebildeten Kurven.

2. *Direkte Erregung der Pedalganglien durch Auflegen eines Kochsalzkristalls.* Auf eine derartige Tonuslösung durch Reiz deuteten einige Resultate bei den in Gemeinschaft mit Frl. C. E. SLUITER unternommenen Versuchen über die Pedalkonstanz der Wiederholungskurven nach Massgabe des Aktivitätszustandes der Ganglien (7, 8). Verglichen wurden Normalkurven (ursprüngliche Kurve und eine oder mehrere der Wiederholungskurven) mit Wiederholungskurven, wenn vor Applikation der Belastung ein Kochsalzkristall mit einer Pinzette vorsichtig auf die Pedalganglien gebracht worden war. Abb. 2A gibt als Normalkurve Wiederholungskurve 2 wieder; nach widerstandsloser tonischer Verkürzung wird ein Kristall appliziert und 5 min später die Bremse des Dehnungsapparates geöffnet. Die Restlast des Registrierhebels, welche zur Streckung des Fadens dient, dehnt den Fuss sofort um etwa 1 cm (*a'—b'*); dann wird die Dehnungslast angehängt und es folgt die Strecke *a''—b''*. Die ganze steile Strecke bis *b''* ist länger als in den Vorversuchen ohne Kochsalz; auf sie folgt ein „Umschlag", welcher nicht zur Konstanz, sondern zu einem Minimum und sodann zu einer tonischen Verkürzung (*c—d*) führt; diese macht neuerlicher Dehnung Platz. Statt Konstanz zeigt sich in dieser Wiederholungskurve dynamischer Tonus. Die ihm entsprechende Kurve sinkt unterhalb des Niveau der Pedalkonstanz um dann über das Niveau des peripheren Tonus (Wiederholungskurve 5) hinaus zu steigen.

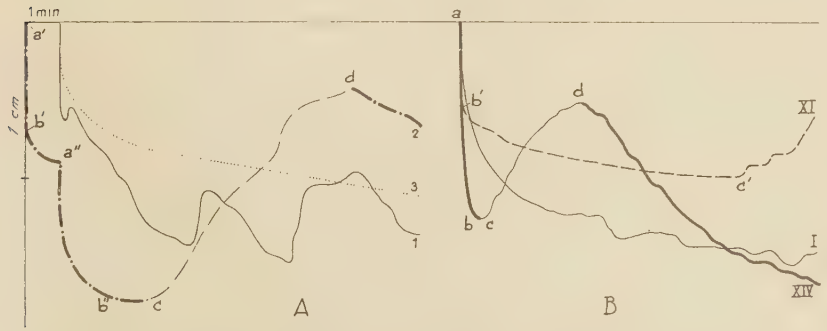

Abb. 2. Dehnungskurven von Füssen, im Zusammenhang mit den Pedalganglien. Versuche mit Applikation von Kochsalz auf die Ganglien.
A. Wiederholungskurven. Objekt *PK–W–KD . A* V, 9-II-'38. Gewicht des Tieres 18 g, Last 10 g. 1. Vorversuch: Normalkurve ohne Kochsalz. 2. Nach Applikation eines Kochsalzkristalles: auffallende Erniedrigung des Tonusspiegels und Dynamik (tonische Kontraktion). 3. Nach Exstirpation der Pedalganglien. *B. Ursprüngliche Kurven.* Gewicht der Tiere 17 g, Last 20 g. I. Normalkurve. XI. Fliesspapierblättchen mit 15 % Kochsalz in Schneckenblut auf die Pedalganglien: Konstanzniveau erhöht. XIV. Kochsalzkristall auf die Ganglien: Tonusabfall und Dynamik (Versuchsserie *PK–R–ZD . B*, 1-V-'41).

Eine ähnliche Dynamik zeigt die ursprüngliche Kurve (= U.K.) nach Auflegung eines Kochsalzkristalls auf die Pedalganglien (Abb. 2B): I, Normale U.K. zur Vergleichung. Die U.K. XIV zeigt nach Applikation von Kochsalz eine sehr steile Senkung der ersten Strecke und einen sehr scharfen Umschlag auf Spannungslänge (*b—c*), woraufhin tonische Verkürzung *c—d* auftritt, auf welche ständige Dehnung folgt. Diese sekundäre

Dehnung überschreitet die Spannungslänge, ohne Tendenz zur Konstanz. Die U.K. XI wurde nach Applikation einer 15% Kochsalzlösung mittels eines damit befeuchteten Fliesspapierblättchens aufgenommen; die sehr steile Senkung ist kürzer, der Umschlag oberhalb der Spannungslänge führt zu einer Konstanz (b'—c'), welche in Verkürzung übergeht, aber viel später als die in Kurve XIV nach Applikation eines Kristalls.

Im Gegensatz zur, durch das Kochsalzkristall ausgelösten dynamischen Reaktion, zeigen die U.K. und W.K. nach Applikation eines Fliesspapierblättchens mit Kochsalzlösung auf die Pedalganglien eine Konstanz und zwar auf höherem Niveau als die Spannungslänge. Dasselbe Resultat bekommt man nach Auflegung eines Blättchens aus kristallinischem Kochsalz.

C. Diskussion der Ergebnisse und Zusammenfassung.

Meine erste zusammenfassende Arbeit auf dem hier behandelten Gebiete war der Wiederherstellung des Tonus (Tonusniveau *und* Tonusspiegel) nach vorhergegangener Dehnung gewidmet. Die Regulierung des Tonus durch das Zentralnervensystem wurde absichtlich ausser Acht gelassen (vergl. 14, S. 27). Jetzt ist die Funktion der Pedalganglien unser Hauptproblem.

Auf Grund seiner Erfahrung an Aplysia (3), bei welchem Objekt sich bislang lediglich Tonusverminderung durch die Pedalganglien hatte nachweisen lassen, hatte JORDAN die genannten Ganglien als einheitliche Zentren der Tonusfunktion aufgefasst. Ihr Einfluss sollte auf der Koordination des Aktivitätszustandes beruhen. Wie sich der tonische Zustand zwischen allen Muskelteilen des normalen Tieres koordiniert, so sollte auch eine Angleichung der Aktivität zwischen Zentren und Peripherie bestehen, wobei die Pedalganglien, da sie als Urfunktion die Tonusverminderung haben, dauernd oder doch vorwiegend im Zustande niederen Aktivitätszustandes verkehren sollten.

JORDAN hat diese Form, die Zentrenwirkung zu beschreiben, auch auf Helix angewandt (4, 5), obwohl er bei diesem Objekt und analog bei Ciona intestinalis, aber im Gegensatz zu Aplysia, auch eine tonussteigernde Wirkung der Pedalganglien fand, nämlich den Zentraltonus (Pedalkonstanz). Um zu zeigen, dass die Richtung des Ausgleiches dem Aktivitätszustand in den Ganglien, verglichen mit demjenigen der Muskeln, entspricht, hat JORDAN eine Reihe von Versuchen gemacht, bei denen er diesen Aktivitätszustand der Ganglien erhöhte (Kochsalz) oder erniedrigte (Kokain). Der Aktivitätszustand in den Muskeln wurde gleichfalls durch Aufbringen von Kochsalz erhöht (7); Erniedrigung des Muskeltonus war möglich durch Kokain oder durch passive Dehnung, welche das Tonusniveau verminderte und parallel damit den tonischen Aktivitätszustand der Muskulatur herabsetzen sollte (4, S. 203; 5, S. 542). So stellte JORDAN der **„eingreifenden"** Regulierung durch Impulse, die **„koordinative"** Regulierung nach dem Prinzip der Isostasie gegenüber (8): *einheitliche Reaktion des Muskels oder des Nervenmuskelsystems.* Sie ist unverträglich mit antagonistischer Muskelinnervation. *Tonuslösung und Tonuserzeugung sind zwei Phasen einer Funktion, die einander bei ein und derselben Erscheinung ausschliessen, also nur nach einander auftreten können.*

Der allgemeine Gesichtspunkt, der mich bei meinen Untersuchungen leitete, war, um am Auffinden einer andern Theorie der zentralen Tonuserzeugung und Tonuslösung bei unserm Objekt mitzuarbeiten. Gewisse Erfahrungen hatten bei uns Zweifel an der Richtigkeit der alten Theorie geweckt. Ich habe nun die direkte Reaktion des Tonusspiegels auf die verschiedenen experimentellen Eingriffe feststellen können und mit den Abänderungen des Niveau der Pedalkonstanz, unter dem Einflusse der gleichen Eingriffe, verglichen. Die Resultate haben unseren Zweifel verstärkt:

1. *Die Reaktion des pedaltonischen Präparates auf Belastungsgrösse*: *a.* Es zeigte sich, dass *hohe Last* die Tonusarmut des Muskels, der in Verbindung mit den Ganglien steht, steigert: Aktivierung der Tonuslösung (steile Strecke der ursprünglichen Kurve; Erniedrigung der Niveaux der Wiederholungskurven). *b. Niedere Last* aktiviert die

Konstanzerzeugung, als spezifisch erregende Funktion der Pedalganglien; in aufeinanderfolgenden Wiederholungskurven steigt das Niveau. Zum Auftreten einer wirklich steilen und langen Anfangsstrecke der Dehnungskurve kommt es bei niederer Belastung nicht. Zwar verläuft dieser Kurventeil etwas steiler als beim ganglienlosen Fuss, wegen des niedrigeren Tonusspiegels des normal innervierten Muskels, aber *es fehlt hierbei der, die Tonuslösung reaktiv verstärkende Reiz der höheren Last.*

2. *Kokain* wird in geringer Konzentration auf die Pedalganglien gebracht. Stets fehlt eine deutliche steile Strecke, dahingegen schwindet die Konstanz nicht, das Niveau wird nur erniedrigt. Das Gift dämpft die Tonuslösung mehr als die Konstanzerzeugung. Man muss hierbei an die *Dämpfung der Wirkung einer spezifisch tonushemmenden Funktion* denken. Diese Auffassung gewinnt an Wahrscheinlichkeit durch unsere Versuche mit Aktivitätserhöhung in den Pedalganglien durch Auflegen von Kochsalz.

3. *Kochsalz* wird als Kristall, oder als Lösung von 15 % angewandt. Nach Auflegen eines *Kochsalzkristalles* tritt *keineswegs lediglich Muskeltonuserhöhung* auf, sondern zuerst namen wir (7) eine ausgesprochene Tonuslösung wahr, die sich in langer und steiler erster Strecke der Dehnungskurve zu erkennen gibt, die niedriger sinkt als das normale Konstanzniveau. *Ein Kristall fördert also auch die Tonuslösung,* erhöht dann durch eine Strecke tonischer Verkürzung das Konstanzniveau. *Kochsalzlösung* zu 15 % oder ein Blättchen aus kristallinischem Kochsalz verursacht keine Tonuslösung (erstre Strecke), sie erhöht lediglich das Konstanzniveau. Vermutlich lokaler Verhältnisse wegen, wird nur die erregende Funktion der Pedalganglien aktiviert durch Applikation von Kochsalz auf eine grössere Oberfläche (Blättchen gegenüber Kristall). Die Wirkung eines kleinen Kristalles lässt die Erklärung zu, dass das Salz erst auf ein hemmendes Zentrum reizend wirkt (weil dieses vermutlich eine niedrigere Reizschwelle hat und die Lokalisation der Einwirkung für dieses Zentrum günstiger ist), um sodann die Konstanzerzeugung zu aktivieren.

Wenn dies alles wahr ist, muss man Tonuslösung auch durch passende Reizung der Pedalganglien und der Nn. pedales erzielen können. Auch das war mir gelungen und zwar durch mechanische Reizung (vgl. auch Mitt. III u. IV).

4. *Mechanische Reizung des Nervus pallialis* (Durchschneidung) im Zustande der Pedalkonstanz verursacht, ähnlich wie Exstirpation der Pedalganglien während dieses Zustandes des Muskels, schnelle Dehnung (Tonuslösung während der Konstanz). Nach dieser Tonuslösung durch Reflexreizung war in unserm Institut lange vergeblich gesucht worden.

5. *Allgemeine Schlüsse aus diesen Ergebnissen*: Die beschriebenen Resultate sind nicht in Einklang mit der Ausgleichhypothese zu bringen. Sie lassen sich dahingegen als die Beeinflussung von *zwei gleichzeitig wirksamen antagonistischen Funktionen* erklären:

a. Kokain wirkt in der Tat auf die Konstanz derart, dass die Wirkung mit dem Ausgleichschema beschrieben werden kann. Allein die erste steile Strecke, die nach diesem Schema steiler und länger werden müsste, wird durch diese Erniedrigung des Aktivitätszustandes unterdrückt. Kochsalz müsste nicht nur das Konstanzniveau erhöhen, sondern auch die Häufigkeit der steilen Strecke vermindern; die Lösung wird aber gefördert. Kokain und Kochsalz wirken also gleichsinnig fördernd auf zwei *einander entgegengesetzte* Funktionen.

b. Auch die Reaktionen auf Belastungsgrösse, wie Aktivierung der Spiegel- oder Konstanzerzeugung durch niedere Last und Aktivierung der Tonuslösung durch hohe Last widersprechen der früher benutzten Beschreibungsweise der Tonusregulierung (vergl. auch 17, S. 108). Beide Reaktionen können nunmehr als ein Reflex beschrieben werden. Das erlaubt mir auch den „Halbtierversuch" (vgl. Mitt. I, S. 1152) anders zu erklären als JORDAN dies tat, da ihm zentrale Tonuslösung durch hohe Last unbekannt war.

Wenn der Einfluss der Last, wie wir ihn durch die Wiederholungskurven kennen (Niveauerniedrigung durch hohe Last, Niveausteigerung durch geringe Last), sich auch im „Halbtierversuch" geltend machen sollte, so ist folgende Erklärung dieses Versuches möglich: die hoch belastete Fusshälfte verursacht reflektorisch zentrale Aufhebung des Tonus in der Gesamtmuskulatur, also auch in der „registrierenden", nicht extra belasteten Hälfte. Die ursprüngliche, erste Dehnungskurve des intakten Fusses zeigt bei hoher Belastung aber nur Spiegelerniedrigung bei gleichem Niveau und wir werden noch kontrollieren müssen, ob diese Spiegelerniedrigung eine Niveauverminderung zufolge hat, die dem Niveauverlust im „Halbtierversuch" entspricht.

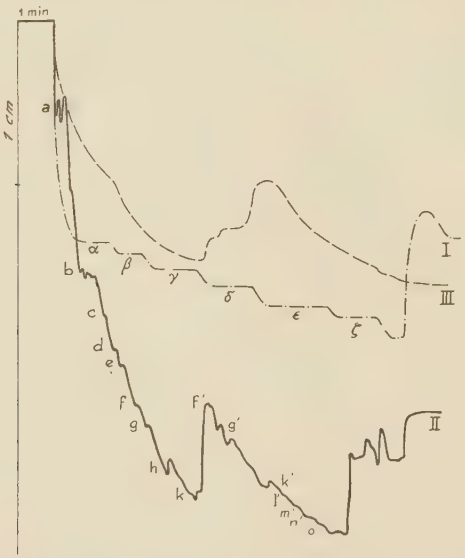

Abb. 3. Ursprüngliche Kurven von pedaltonischen Objekten (Last bei I und II, 20 g). Kurve II zeigt bei *a, b,* u.s.w. einen Kampf zwischen Tonuserzeugung und Tonuslösung. I. Jedesmal nach kurzdauernder Konstanz (α, β, u.s.w.) plötzlicher Abfall, welcher auf das Eingreifen eines hemmenden Impulses hinweist. III. Kurve ohne steile Strecke, weil die Last niedrig ist (10 g).

Ich habe früher (14, S. 71) beschrieben, und immer wieder wahrgenommen, dass in vielen Fällen während der pedaltonischen Dehnungskurve sozusagen ein „Kampf" zwischen der Tendenz zur Konstanzbildung und derjenigen der Tonuslösung auftritt. Die Dehnung wird plötzlich unterbrochen, scheinbar tritt Pedalkonstanz auf, plötzlich aber wird die steile Dehnung fortgesetzt. Dieser Wechsel kann sich im Falle sehr ausgesprochenen Kampfes öfters wiederholen (Abb. 3, Kurve II). Ich habe damals an die Möglichkeit gedacht, dass die Tendenz zur Tonuslösung in den Pedalganglien, die Tendenz zur Konstanzerzeugung dahingegen in der Fussmuskulatur selbst lokalisiert sei. Da es aber offenbar ein tonuserzeugendes Zentrum in den Pedalganglein gibt, dürfte kaum daran zu zweifeln sein, dass es neben Netzkonstanz auch eine Pedalkonstanz gibt. Der plötzliche Abfall der Kurve I, nach kurzdauernder Konstanz, weist auf alle Fälle auf das Eingreifen eines hemmenden Impulses hin.

Alle diese Resultate und diejenigen gemeinsamer Versuche (7) wurden Anlass zu weiteren Untersuchungen, deren Aufgabe es war, die vermutlich vorhandenen zwei Zentren in den Pedalganglien zu finden, über die in der nächsten Mitteilung III berichtet werden wird.

Die übrigen Resultate lassen das Prinzip des Ausgleiches ausser Diskussion. Sie ergaben dass die Muskelviskosität nur einen Teil der Tonuserscheinungen zu erklären imstande ist, was in unserem Institut auch immer ausgesprochen worden ist (6, 10, 11, 12, 13, 14). Obwohl Tonus und selbst die Konstanz (nach Maas, 9) keinen Anspruch an den Stoffwechsel stellen, muss beiden eine reflektorische Grundlage zuerkannt werden. In meinen Versuche spielte Viskosität als solche eine überwiegende Rolle nur in folgenden Ergebnissen: Kälte unter 12° erhöht den tonischen Widerstand der ersten Dehnungsstrecke, während Wärme (oberhalb 27°) ihn erniedrigt. Auch Wassermangel erhöht den Widerstand, während Sättigung des Muskels mit Wasser merkwürdigerweise nicht den umgekehrten Effekt hat.

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Medicine. — *Classification of the disorders of language, speaking and voice.* By F. GREWEL. (Communicated by Prof. A. DE KLEYN.)

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The doctrine of the disturbances mentioned in the title, which used to be termed disorders of speech, has developed slowly; and only gradually has a classification into different large groups come into existence; the development in this respect has not yet come to an end. The deepening of our knowledge in this field was difficult, however, the more so because the material for this greater knowledge had to be drawn from various other domains of science, most of which have just begun to develop. Thus our knowledge of the neurological diseases attended with disorders of language and speaking has gradually increased, while also the pathological-anatomical basis of the diseases which affect voice and speech has been studied more thoroughly. One gets a more and more profound insight into the physiology of voice and speaking, so that experimental phonetics have become a separate subject of study. In many respects we have only just begun to apply their results to the pathological physiology of voice and speaking, a field which, it is true, does not lie fallow, but can still be exploited more extensively; here there are perhaps also hidden possibilities to give to linguists a better insight into certain changes of language. The development of speaking in childhood has become a subject of study and the knowledge of the development of the child's language has also become greater. Besides the knowledge of the disorders of language among patients has also deepened; the theory of aphasic disorders especially has developed to an enormous extent. Medical investigators have realised that one cannot be satisfied with a naive opinion about language defects among such patients; ever since PICK's investigations and again just lately the insight that the disturbances of aphasic patients should also be studied from the linguistic point of view has been roused. It goes without saying that the psychological study of these patients, who are hampered or disturbed in fundamentally human functions, such as language and speaking, yields also important results.

So we are provided with an abundance of data the arranging of which is very difficult; it follows indeed clearly from the above that no expert can any longer be master of the whole field.

The diseases in the domain of human acts of language and speech have up to now been classified as disorders of speech and disorders of voice. A new arrangement based on newer insights is necessary, however, and, as we think, possible too. In making it, one should start from the insight that man has the power of *speech*. He can use this power by making himself understood in a system of conventional symbols, his *language*. It is only in a language that the human power of speech gets its great biological realisation and its great social significance [1]). So the power of speech is a disposition for which symbols of language are necessary; the disposition is every time realised in the *act of speaking*, the *speaking*. So one must make a clear distinction between diseases which make the language of the patient disappear or hamper it and those where the act of speaking is made impossible or difficult. It is true that for a long time these two groups of disorders have been set over against each other, but again and again such disorders which concern the incapacity for the act of speaking threaten to be introduced within the group of aphasic disorders. One can even say that the prototype of aphasic conditions, the group of patients-described by BROCA who had lost "le souvenir du procédé qu'il faut suivre pour

[1]) J. VENDRYES. Le langage. Paris 1921.
G. RÉVÈSZ. Die Sprache. Proc. Ned. Akad. v. Wetensch., Amsterdam, XLIII, 1940.

articuler les mots" (BROCA) do not belong to the aphasias in a narrower sense, but at the most form an intermediary group between aphasia and dysarthria.

For gradually it has become clear that, at least for the expressive aphasias, the incapacity for language is the most important phenomenon [2]). It is true that this is made more complicated by a whole series of psychological phenomena and by radical disorders in the whole power of expression [3]), so of a fundamental field of the praxis; but for the classification it is not the psychical and psychomotoric attendant phenomena that are important, but the essential fact that not the act of speaking, but the language has suffered.

Considering what GUTZMANN [4]) in his monography ranges under dysarthria one can only conclude that his classification is entirely obsolete. Stammering- and stuttering-defects and the disabilities of speaking caused by mechanical defects in the articulation-apparatus are all styled as forms of dysarthria. The instrumental investigation of these defects and disorders has made it possible to establish such fundamental differences that already ZWAARDEMAKER, QUIX, TEN CATE, later SCRIPTURE, DE SANCTIS could distinguish nervous disorders of speech, such as stuttering, from organic ones. It appears to be necessary to indicate clearly that by dysarthrias one understands disorders in the act of speaking caused by lesions of the nervous system, except for the cerebral cortex, in other words as a consequence of subcortical or peripheral neurological injuries. By this the field of the dysarthrias is greatly narrowed down, but on the other hand this definition of dysarthria does include the enlargement that appeared necessary in the long run. Thus for instance that which was clearly indicated by LEYSER [5]) when he considered the disorders of speech caused by affections of the basal nuclei, such as post-encephalitic parkinsonism and athethosis, as dysarthrias. For by this it has not only been confirmed that besides the disorders of articulation and of tempo phenomena of phonation and respiration can also occur in dysarthria, but it has, moreover, become clear that the disturbances of melody, the iterations, the changes of tonus, etc. in speaking are deserving of our attention.

Both the nervous disorders of the act of speaking and the mechanical ones form groups separate from the dysarthrias.

Considerations that are essentially the same as for the abnormalities of language and of the nervous, mechanical and neurological disorders of the act of speaking among adults also have their value for the child; on the understanding, however, that here individuals, so also language and speaking are affected in their *development*. So the problem changes in so far that besides the abnormalities found among adults, hindrances in the development of language and speaking are found as well. In these disturbances of growth one must again distinguish two groups: the disorders of tempo, where the unfolding of a power only takes place more slowly and the qualitative disabilities.

In the same way as among adults a defect in the articulation-apparatus can lead to articulation-defects; this is possible with the child, moreover, on account of an incomplete mastering, i.e. of a function of this apparatus as yet incomplete. As is known the disorders of the tempo of development play a great part especially in the domain of speaking.

When one separates the voice-defects from the speech-defects one must point to the fact that this obtains only when it is the voice alone that makes us hear abnormalities, while for the rest speaking is intact. Disorders of the voice namely repeatedly occur in cases of disability in speaking, especially in dysarthrias; in such cases one considers them as belonging to the dysarthric syndrome.

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[2]) H. HEAD. Aphasia and Kindred Disorders of Speech. Cambridge 1926.
M. A. v. MELLE. Over Aphasie. Acad. Proefschrift, Amsterdam 1900.

[3]) F. GREWEL. Taalpsychologische problemen bij aphasie. Alg. Nederl. Tijdschr. v. Wijsbegeerte en Psychologie, XXXIV, 1941.

[4]) H. GUTZMANN. Die dysarthrischen Sprachstörungen. Wien u. Leipzig 1911.

[5]) E. LEYSER. Ueber einige Formen von dysarthrischen Sprachstörungen u.s.w. Zeitschr. f. d. ges. Neur. u. Psych., LXXXXVIII, 1924.

What has been explained above leads us to make the following classification:

A. THE GENERAL ABNORMALITIES.

I. *Disorders of language-functions.*

a. *Aphasia.* There is incapacity of using language, while speaking is essentially undisturbed. In aphasias very definite parts of the cerebral cortex are injured and dependent on the place of the lesion one can distinguish certain types of aphasia, namely expressive, receptive, amnesic and perhaps central aphasia. As the aphasias are disorders in the use of the instruments of language, it will appear useful to give a linguistic basis to the analysis of the aphasic disturbances; one will have to look for the abnormalities in the structural laws of the language, not only phonematical (TRUBETZKOY), but also syntactical ones, etc. [6]).

b. PICK's disease certainly fits in with the aphasias, also anatomically [7]). It is characterised by a gradually progressive incapacity for language and by its complication with other disorders, particularly presenile dementia, loss of initiative, reduction of mental productivity and deterioration of the practognosis (GRÜNBAUM).

c. Several diseases that are attended with language-disorders show anatomically conspicuous lesions of the cerebral cortex. These alterations of the cortex must be considered as more or less important in ALZHEIMER's disease, with some forms of palilalia and with language-disorders in senile dementia. Anatomical lesions of the basal nuclei, however, which are also present in PICK's disease, are found especially in the two former clinical pictures.

II. *Disorders of the function of speaking.*

a. *Dysarthria.* This term serves to summarize the disorders in the mechanism of speaking, as a consequence of subcortical or peripheral lesions of the nervous system. Here one finds, dependent on the localisation in the nervous system, a great number of different neurological syndromes, in each of which the acts of speaking are impaired, while the language is intact. The disability of speaking expresses itself in disorders of articulation, phonation and respiration, in tempo and modulation; either in all these domains or in some of them. The hindrance of the mechanism of speaking can be explained neurologically; again and again different disorders occur in accordance with the parts of the nervous system affected; so a classification of the dysarthrias based on the localisation is possible and gradually this is being made. The bulbar and pseudo-bulbar, the cerebellar, choreatic, athethotic and post-encephalitic forms of dysarthria can be mentioned in this connection [8]).

b. *Dyslalia.* The dyslalias too are the consequence of technical disorders, but only of the articulation-mechanism. They are due to anatomical defects in the peripheral apparatus itself; cleft palate, abnormalities of the tongue, etc. One also calls them peripheral-mechanical disorders. Disability in speaking caused by peripheral nerve-lesions such as neuritis (paralysis of the palate after diphtheria; hypoglossusneuritis, etc.) is classed among the dysarthrias. The hampered speaking found with myasthenia gravis can be looked upon as intermediary; it is best to group it with the dyslalias.

c. *Aphemia and dysphemia.* By these one understands the large group of disorders of speaking on a neurotic basis, also among the constitutionally nervous and inferior, such

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[6]) R. JAKOBSON. Kindersprache, Aphasie und allgemeine Lautgesetze. Sprakvetenskapliga Sällskapets i Uppsala Förhandlingar. 1940—1942.

F. GREWEL. L.c.; and Problemen bij de aphasiestudie. Psychiatr. en Neurol. Bladen, XXXV, 1941.

[7]) C. VAN DER HEIDE. Klinisch-anatomische studie over Picksche ziekte. A'dam, 1934.

[8]) M. NADOLECZNY. Kurzes Lehrbuch der Sprach- und Stimmheilkunde. Leipzig, 1926.

J. EUZIÈRE, J. TERRACOL et R. LAFFON. Les troubles de la parole dans les affections du système nerveux central. Revue franç. de Phoniatrie, 1939.

as the feeble-minded. They are the defects of stuttering, clottering and stammering. Further subdivisions are necessary within this group; there are also transitions, as between stuttering and clottering. Further there are differences in stuttering as e.g. between the stutter-neurosis and the hysterical stutter.

III. *Disorders in the function of the voice.*

Here there is the same classification as in the preceding group.

a. *Neurological disorders* of the voice, as with tabes dorsalis, posticus-paralysis, etc.

b. *Organic disorders* of the voice through local affections, as hoarseness as a consequence of a tumour of the larynx or inflammation of the vocal chords, etc.

c. *Neurotic (nervous) disorders* of the voice, as phonasthenia (as a rule), hysterical hoarseness or aphonia, etc.

IV. *The peripheral-impressive disorders* of speaking form a separate group. The changed speaking attending deafness is due to the fact that deaf people lack an acoustic control of their own speaking-production, articulation, phonation, melody and accent deviating so much that the speaking changes in character.

B. THE ABNORMALITIES IN THE DEVELOPMENT.

As has been explained above, there are among the young, besides the disorders which the adult knows, the disorders in the development, as a separate group of affections. These disorders in the development of human speech-utterances must be studied separately. Here again it appears to be useful to distinguish the language-function and the function of speaking; then in the one case one meets with difficulties which the child has to overcome when it learns how to master the language, i.e. the whole of the symbol-conventions, in the other case in acquiring sufficient readiness of speaking. This does not mean that both develop entirely independant of each other; it seems to us namely that readiness of speaking also has some influence on the acquiring of language; but this is beyond the scope of this article.

The mastering of the language has two sides. In the first place a sufficient intellect is necessary for this; if this is wanting no language or a very limited language develops; the mutitas idiotica and the poor language of many feeble-minded, therefore are not among the disorders of language in a narrower sense, though their study contributes much to our understanding of the development of language among normal people. There is, however, a second side to the development of the language in childhood; the power to handle the conventional symbols namely has a development which is more or less *in*dependent of the intelligence. Thus one sees intelligent children learning the mastery of a language with difficulty and on the other hand feebleminded among whom the fluent mastering of the language develops quickly (verbal giftedness). Though this may not be profound in such a case and may cause a spurious pseudo-intelligence to arise, essentially it cannot be denied. (And it is curious that a language, i.e. a system of conventional symbols forms yet again such a powerful means to the drawing of conclusions that many a verbally gifted dunce gets along fairly well by means of the fluency of his language.) Besides, however, there are defects in the development that only concern a part of the mastery of the (a) language.

These considerations lead us to make the following classification:

I. *Disorders in the language-development.*

a. *Congenital acoustic agnosia.* This concerns not the group of children who suffer from congenital psychic deafness and who cannot be "roused" acoustically, so do not react on a single sound-stimulant, but the group of the congenitally word-deaf. These children do react on sound and are not deaf to the sounds of the human voice; nor are they feebleminded; yet they do not understand or produce language. The clinical picture

of congenital word-deafness, which has always remained inexplicable, suddenly becomes clear when one keeps in mind that it must not be regarded as a congenital receptive disorder, but that it is an arrearage of the power to handle language-symbols.

b. Some *forms of hearing-muteness,* where it is especially the active use of language that is attended with difficulties, while the language heard is understood relatively well, can perhaps be included within the group of disorders in the language-development.

c. *Congenital agrammatism,* first described by LIEBMANN, also becomes clearer through this classification; here there is, with sufficient intelligence, a partial hampering of the language-development, namely that of the grammatical-syntactical functions (symbols).

d. *Belated recognition of a phonological opposition within* a correlation-pair is a phenomenon that can repeatedly be observed in children. By this one should understand the incapacity to distinguish phonemes that belong together, such as *d* and *t*, *b* and *p*, and possible others, within the language. Such a child does not only *say* e.g. "toe te teur tan toch ticht", but also writes it. The wrong writing shows that here it is not a disorder of speaking — up to now this phenomenon was taken to be that — but a disorder of the discriminative powers within the language-domain, namely of the phonological oppositions. The description given here of this disorder shows that it is necessary to make use of modern linguistical conceptions to get a better insight in the nature of these disabilities. In this case the conception of the phonological opposition within the meaning of TRUBETZKOY and his school had to be used [9]).

e. *Congenital aphonemia.* FONTES [10]) has described an isolated congenital (and hereditary) disorder in the use of the *r*; he considers this an isolated congenital aphasia. To me it seems right to regard it as an isolated disorder of the language function, but as it only concerns a single phoneme it seems better to call it congenital aphonemia.

f. *Echolalia* in childhood, of long duration should perhaps be included within this group; it occurs especially among the imbeciles.

II. *Disorders in the development of speaking.*

a. *Hearing-muteness* is generally due to the incapacity of learning how to speak or to the fact that this is attended with great difficulties. The little patients can indicate the rhythm of a word, but fail in speaking, often even in pronouncing a combination of consonants. One can distinguish various forms of hearing-muteness, but this same principle obtains for all of them. They are all due to a motoric infantilism.

b. *Belated development of speaking.* Instead of in the second year speaking develops some years later. In such cases a language-factor sometimes plays a part as well; but there are children who understand every thing clearly, try to speak and fail in this. Here too, one can always establish that there are arrears in psychomotorics [11]).

c. *Stammering-defects in youth: developmental dyslalia.* Many children find it difficult for a longer period than is normal to pronounce one or two phonemes. Every child shows disorders of articulation, especially of many consonants for a certain physiological period; this is the physiological stammer. It is due to a technical incapacity, as a consequence of the insufficient suppleness and promptness of the articulation-apparatuses to produce the desired sounds in the right manner. So with the young child one phoneme represents itself and some others. The other phenomena that attend the undeveloped speaking are irrelevant in this connection. A normal child overcomes this within some years, at any rate before the school-age. This remark implies that abnormal children, e.g. mentally defectives, are less successful in getting over this

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[9]) N. S. TRUBETZKOY. Grundzüge der Phonologie. Prague 1939.

[10]) V. FONTES. Sur la signification fonctionelle de certaines altérations du langage. Zeitschr. f. Kinderpsychiatrie, VII, 1941.

[11]) F. GREWEL. De psychomotoriek bij ontwikkelingsstoornissen van de spraak. Maandschr. v. Kindergeneeskunde, X, 1940.

mechanical incapacity (frequent dyslalia among the feeble-minded) and that the persisting of a stammer-defect in a constitutionally normal child mostly points to a development disorder, particularly to a children's neurosis.

d. *Clottering and stuttering* frequently occur during the development, especially during the period when speaking cannot yet follow thought (clottering) or when the child has not yet enough language-material to satisfy the existing impulse to comminucate (stuttering). This physiological clottering and stuttering are overcome and it is only through neurotic (or neurological) causes that the stutter can persist [12]). Only in the puberal period, when the motorics pass through a temporary desintegration before reaching their definite organisation stuttering may perhaps again occur physiologically, or on a physiological basis.

e. So already in youth aphemias and dysphemias can develop from the physiological stammer, clotter and stutter, under the influence of neurotic mechanisms, or, as in the case of stammering insufficient training. Children's neuroses cannot be regarded as identical with those of adults; therefore there is some sense in mentioning these children's aphemias besides the aphemias of adults. Children's neuroses are often overcome spontaneously; this also obtains for the neurotic disorders as regards their speaking. Perhaps one should take together the groups II c and d of the development-disorders of speaking as one group.

III. *Disorders in the development of the voice.*

a. The *mutation-disorders* of the voice are organic: cracking of the voice, a voice that is too deep or falsetto-voice. Here it is often difficult to say where lies the border between the physiological and the pathological domain. They are certainly pathological with pubertas praecox or on the other hand with hypopituitary infantilism, or mutation delayed in another way.

b. *Technical disorders* in speaking cause the hoarseness that is so frequent in youth.

c. *Psychogenic disorders,* such as whispering in youth, are sometimes found in smaller children (timiditas infantum).

IV. *Peripheral-impressive disorders.*

These play a great part in certain disorders of the development of language and speaking. The various forms of deafness and hardness of hearing can cause insufficient speaking or can account for the fact that it is wanting. It goes without saying that in serious cases, when no other methods are resorted to, *language* is not acquired either. The anatomical defect can be localised in the internal ear or in the central connexions up to the cerebral cortex (bilaterally).

It is clear that several of the above-mentioned groups of disorders need a further subdivision; this obtains e.g. for the dysarthrias and developmental dyslalias; these subdivisions have already begun to be worked out. We do not mean, however, to enter into these questions of practical diagnostics.

Because several of the disorders dealt with here can occur together, in certain concrete cases the classification may seem to be difficult. Thus a hemiplegic aphasic person can at the same time be dysarthric. Other cases are intermediary, so that one can speak of a mixed case or a transitional form. Thus modern investigations teach us that the subcortical motoric aphasia of DÉJERINE, wrongly called anarthria by PIERRE MARIE, must be considered as a dysarthria; the p h o n e t i c a l deterioration which one finds with this disorder must be considered as a consequence of the dysarthria. JAKOBSON (l.c.) wrongly uses *these* data to argue that a p h o n o l o g i c a l deterioration occurs in aphasia, though this in itself cannot be denied. Pure aphasia-material, however, should furnish the evidence. In subcortical motoric aphasia, besides the phonetical disorder, one

12) F. GREWEL. Stottern. Psychiatrische en Neurologische Bladen, XLI, 1937.

meets with agrammatism and a limited vocabulary as well; and so in this case there certainly is a deterioration of language besides the phonetical defect. So here one must speak of an intermediary affection: there are both dysarthric and aphasic phenomena [13]).

A third question is that a disorder will now have to be considered as belonging to one, then to another group. Thus stuttering should mostly be considered as an aphemia, but there are cases of organic stuttering which must be regarded as a dysarthria. Moreover there exists perhaps an aphasic stutter as well. — A developmental dyslalia may have been fixed later in life and must then be considered as a dysphemia.

From all this it appears that the classification of the disorders in the use of language, speaking and voice given here cannot be applied schematically; but it seems to us that this scheme essentially arranges and summarizes our present knowledge of the field under consideration. This obtains especially for the abnormalities in the language- and speaking-domain of the child. Instead of a heterogenous group of affections one finds here, besides the disorders that also occur among adults, a series of disorders in growth, either of the tempo of development (so-called tempo-disorders: B Ia, b and c, B IIb), or of a qualitative nature.

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[13]) F. GREWEL. Review of: Alajouanine, Ombrédane et Durand, La desintégration phonétique. Logopaedie en Phoniatrie, XIV, 1942.

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